TECHNICAL ELECTRICITY

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PRINCIPLES AND PRACTICE

BY

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PREFACE

THIS book is intended primarily to provide an introductory course in Electricity and Magnetism for students in Technical Schools and Colleges. The subject matter covers at least the syllabuses of the first examinations of the City and Guilds of London Institute, Union of Educational Institutions, Union of Lancashire and Cheshire Institutes, Northern Counties Technical Examinations Council and of similar bodies, and, moreover, should provide an adequate preparation for a student proceeding to higher grade work. The practical aspect of the subject has been emphasised throughout, as it is of particular importance to those who are about to enter, or are already engaged in, industrial work. It is tacitly assumed that the reader is acquainted with the rudiments of mechanics and heat.

The rapid increase of scientific knowledge in recent years has been especially marked in the subject under review, and in consequence no small difficulty arises in the selection of material, having regard to the restricted size of this volume. However, the authors have endeavoured to include as many technical improvements and discoveries as possible; and while the details given are necessarily brief, it is hoped that the descriptions are sufficiently clear for the general principles to be easily understood. Many students do not fully appreciate the theory and performance of the instruments they handle, and particular attention is directed therefore to Chapter X.

The text is supplemented by means of a number of worked-out examples and numerous line diagrams. The latter have been drawn specially for the book, and by reason of their simplicity are easily capable of reproduction by the student. A selection of suitable questions is appended to each chapter, and the majority of these have been taken from actual examination papers, by the courteous permission of the Examining Bodies enumerated above. In the Appendix will be found a useful summary of units, and also data relating to the electrical and magnetic properties of various materials.

We are deeply grateful to Dr. N. W. Robinson for his reading and criticism of the manuscript; and our thanks are due to the Publishers for their helpful co-operation. The blocks for Figs. 54 and 141 were kindly loaned by Hellesens Ltd. and The British Thomson-Houston Co. Ltd. respectively.

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Technical Electricity

CHAPTER I

MAGNETISM

INTRODUCTORY.—The fact that a certain mineral ore attracted iron filings to itself was known to the Greeks and Romans in the pre-Christian era. This ore was an oxide of iron ($Fe_3 O_4$) called magnetite, and it was first found in quantity in Magnesia, a province of Asia Minor. A substance such as magnetite is referred to as a **natural magnet,** and is to be distinguished from an **artificial magnet,** which is produced artificially from an unmagnetised substance.

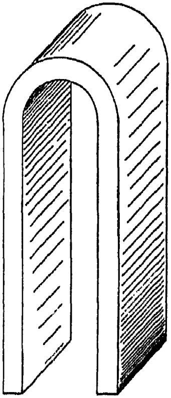


FIG. 1.

ARTIFICIAL MAGNETS.—In the year 1845 the renowned English physicist, Michael Faraday, showed that all substances are capable of being magnetised. Prior to that time it was thought that magnets could be made only from iron and steel, the latter metal being used for the manufacture of **permanent** magnets. The choice of steel for this purpose, in preference to soft iron, is due to the fact that the steel, although less easily magnetised, retains its magnetism to a far greater extent than the iron does after magnetisation.

Artificial magnets are of three kinds, viz. bar-magnets (Fig. 3), horseshoe magnets (Fig. 1) and electro-magnets (Fig. 2). Bar-magnets and horseshoe magnets are permanent magnets and are made of hard steel or of some alloy which has the power of retaining magnetism. The electro-magnet (Fig. 2), on the contrary, is not a permanent magnet. In this case the metal used is soft iron, and the magnetisation is produced by means of an electric current which is passed through an insulated coil of wire wound round the iron. It is found that when the current is stopped the soft iron loses most of its magnetism, so that the electro-magnet functions only when the current is passing through the wire. The electro-magnet is of considerable theoretical interest as its action reveals the intimate relationship that exists between magnetism and the electric current.

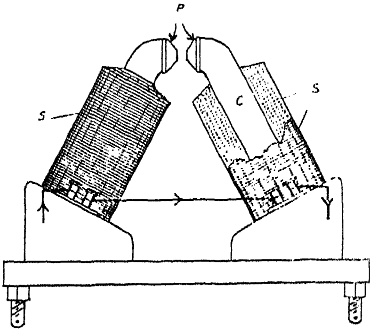


FIG. 2.

PROPERTIES OF MAGNETS.—In a natural magnet the attracting power is found to be strongest at two regions which are on opposite sides of the mineral. The centres of these regions of maximum magnetic attraction are known as the **poles** of the magnet. Furthermore it was known at least as far back as the thirteenth century that if the magnetic stone was suspended, so as to move freely in a horizontal plane, then it always came to rest in such a position that the line joining the poles pointed approximately towards the geographical north and south poles of the earth. This fact was obviously important to mariners, and it was on this account that the magnetic stone was called a leading-stone or **lodestone.**

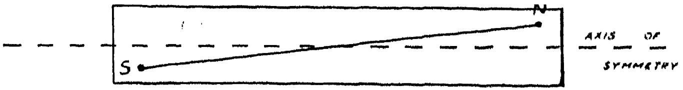


FIG. 3.

Likewise if a bar-magnet is suspended or pivoted, so that it can swing freely, it always comes to rest in the same position, when at a given place. The straight line joining the poles, called the **magnetic axis** of the magnet, points in a direction which is approximately north and south. As in the case of the natural magnet the phenomenon is due to the fact that the earth itself acts as a magnet, in which one magnetic pole is near its geographical north pole and the other near its geographical south pole. The pole of the magnet that points towards the magnetic north pole of the earth is called its north pole, while the other pole is called its south pole.

The vertical plane in which the magnetic axis of a suspended magnet sets itself at any place is called the **magnetic meridian** at that place.

It should be noted that the magnetic axis SN (Fig. 3) of a bar-magnet may not necessarily coincide with its geometric axis as it depends upon the direction in which the magnet is magnetised.

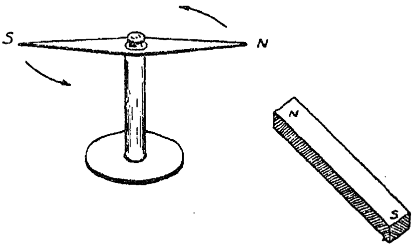


FIG. 4.

Experiment.—Suspend or pivot a small magnetic needle and allow it to swing freely until it comes to rest (Fig. 4). Its axis now lies in the magnetic meridian. Bring the north pole of a bar-magnet *slowly* up to the north pole of the needle. It is found that the latter is repelled. Next gradually approach the north pole of the needle with the south pole of the magnet. It is found that attraction now takes place. The experiment shows that **Like poles repel, unlike poles attract.** This is a fundamental law of magnetism.

If the case of a magnet, which is suspended and lies in the magnetic meridian, is now considered in the light of this law, it becomes evident that the pole of the magnetic needle which points north must be unlike the north pole of the earth. The north pole of the magnet (or needle) is so-called because it is a *north-seeking* pole. Likewise its south pole is a *south-seeking* pole.

INDUCED MAGNETISM. *Experiment.*—Fix a bar of unmagnetised soft iron vertically in a clamp and place it so that the lower end is near some iron filings contained in a watch glass. Next bring a magnet up to the bar and hold it so that the north pole of the magnet is immediately above the upper end of the bar (Fig. 5). The filings will be found to cluster around the lower end of the bar and the tuft of filings will increase in size as the magnet is brought nearer to the bar.

If, next, a small compass needle is brought near the lower end, it is found that the north pole of the needle is repelled by it. This indicates that the lower end has acquired a north polarity. In a similar way it can be shown that the upper end has a south polarity.

The iron bar is said to be magnetised by **induction,** and the rule governing this induced polarity is as follows: **The end of the iron bar nearest to the inducing pole acquires polarity of opposite sign to that of the inducing pole.**

This magnetisation of the iron bar is only *temporary*, for if the magnet is withdrawn from the neighbourhood of the bar it is found that most of the filings drop off.

Experiment.—Replace the bar of iron used in the preceding experiment with a bar of steel and proceed as before. It will be found that fewer filings adhere to the bar in this case. This is because steel does not become magnetised as easily as iron, i.e. it is said to be less **susceptible** than iron. If the magnet is removed it is found that a smaller proportion of the filings drop off than was the case when soft iron was used. On this account steel has a higher **retentivity** than soft iron. Reference has already been made to this fact (page 1).

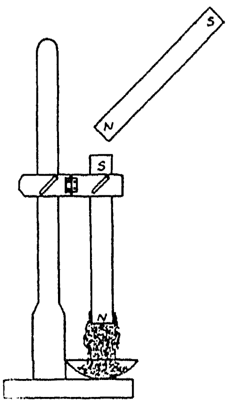


Fig.

The *susceptibility* of a magnetic substance is its power of acquiring magnetism.

The magnetic *retentivity* of a substance is its power to retain the acquired magnetism after the removal of the magnetising influence.

Iron is more susceptible than steel, but is less retentive.

The preceding statement should be qualified by remarking that if the iron is carefully protected from shock its retentivity may be quite large. It is quite correct to say, however, that the **coercivity,** *which is the power of retaining magnetism whatever the subsequent treatment,* is much greater for steel than iron.

Test for a Magnetic Body.—It is now possible to summarise the necessary tests as to whether a substance is a magnet or otherwise. If the material, e.g. paper or copper, is only feebly magnetic, then even small fragments of the substance will not be attracted by a bar-magnet. On the other hand an unmagnetised iron bar will attract either pole of a compass needle as a consequence of the law of magnetic induction, but *one* of the poles of a magnetised body must *repel*, say, the north pole of the needle. Hence **Repulsion is the only certain test of polarity.** Furthermore, it is now evident why in Expt. on page 3, particular emphasis was made with regard to the *gradual* approach of the magnet towards the needle. If the magnet is very strong its inducing action on the needle may suffice to reverse the polarity of the latter, so that attraction ensues even when the poles of the same sign are brought together.

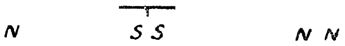


Fig. 6.

Consequent Poles.—A bar-magnet has normally a north and a south pole located near its ends. If, however, a bar-magnet is touched at various points by a pole of another magnet, additional poles may be formed at different points along its length (Fig. 6). The same effect may be produced by reversing at any point the direction of the winding of a wire which surrounds the magnet and carries the magnetising current. Such poles are known as **consequent poles** and their existence is liable to cause trouble and confusion in experimental work, so that every care should be taken to prevent their development in a magnet. Hence magnets should never be indiscriminately brought into contact with each other.

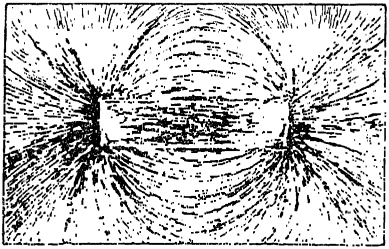


Fig. 7.

Fields of Force.—If a sheet of drawing-paper is placed over a bar-magnet and iron filings are sprinkled lightly over the part of the paper which covers the magnet, the filings arrange themselves along well-defined lines (Fig. 7). These lines are called **lines of force**. The area over which the influence of the magnet is detectable is apparent on observing the distribution of these lines, and this space is called the **magnetic field.** Its extent depends upon the strength of the magnet and upon the sensitivity of the detector, but theoretically, of course, it stretches to *infinity*, i.e. to extremely large distances.

It should be observed that although the lines of force can be plotted only in the plane in which the paper rests, they exist in all the space around the magnet, i.e. above and beneath the magnet. The student should also note that the action of the iron filing as a detector is dependent upon the inducing action of the magnet, whose field is being investigated. The repulsion and attraction between the poles of the magnet and the induced poles in the filing cause the latter to set along the line of force at that particular point, just as a pivoted needle would do.

The conception of the lines of force is due to the fertile mind of Michael Faraday to whom reference has already been made (page 1). Faraday (1791-1867) was a native of London and started life as an errand boy at a bookstore, but before he died he had become Professor of Chemistry at the Royal Institution. He was undoubtedly the leading experimentalist of his age, and one of the greatest scientists of all time. In the course of his investigations on magnetism the question arose in his mind as to how one magnetic pole could act on another which was not in contact with it. In order to answer this problem, Faraday considered that the medium (or *ether*) in between the poles was in a state of strain, and he postulated the existence of tubes or lines of magnetic force in this ether. It is conventional to assume that the lines originate in the north pole of the magnet and terminate in the south pole. Thus the positive direction of the lines in the external space is always to be taken from the north to the south pole.

It will be seen later that it is impossible to isolate a magnetic pole, but if one could conceive a free north pole being liberated in a magnetic field, it would travel along a line of force towards the south pole. In other words, *an isolated north pole, free to move, will travel in the direction of the field.*

Experiment.—In this experiment use is made of the Robison ball-ended magnet which consists of a thin rod of steel (about 25 cm. long) fitting tightly into holes drilled in two steel balls. When this magnet is carefully magnetised it can be regarded as equivalent to two point poles which are situated at the centres of the balls. The magnet is fitted into a cork as in Fig. 8, so that the north pole floats just above the surface of water contained in a deep dish. At the same height above the surface and near the edge of the dish a bar-magnet is fixed horizontally, so that the force on the floating magnet will be approximately the same as that which would be exerted on a free north pole. By placing the cork at various places near a pole of the magnet, trace out the form and direction of the lines of force due to the latter.

The idea of lines of force has proved a very fruitful one both in magnetic and electrical investigations, and a great number of phenomena in connection with these sciences can be explained by investing the lines of force with two definite properties. In the first place, it is assumed that there is a tension along the length of a line of force, that is, it always behaves like a stretched elastic band and tends to contract. Secondly, the lines of force repel each other laterally, as can be observed in the tendency of the lines to bulge outwards from the central portion of a magnet.

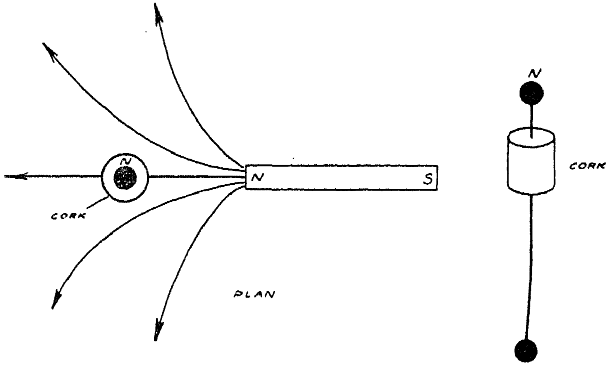


FIG. 8.

Experiment.—To plot lines of force by means of a small compass needle. In this method of plotting, the needle is placed at any point in the field and allowed to come to rest. A mark is then made opposite each end and the compass is moved until the end *s* is exactly over the mark which has just been made opposite the end *n* and the new position of the latter *n′* is marked. The procedure is repeated until the particular line of force is completed, when a *smooth* curve is drawn through the various points obtained.

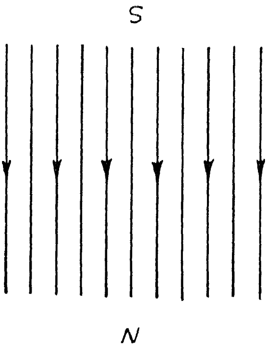


Fig. 9.

(*a*) Fix a sheet of paper on a horizontal drawing-board and use the compass needle to plot the earth's horizontal field. care has been taken to remove any iron in the immediate neighbourhood, then a series of parallel lines will be obtained (Fig. 9).

(*b*) Set up a ball-ended magnet vertically in a *wooden* stand and fix a card horizontally above the upper pole. Trace out the lines of force in its neighbourhood by means of the compass needle. The magnetic field in the plane of the card will be approximately the resultant of the earth's horizontal field and the field due to the upper pole of the magnet. The field of the latter alone, if a *north* pole, consists of lines radiating *outwards* from the pole in all directions. It is evident that at a point such as P (Fig. 10) the two fields will assist each other, but they will be in opposition on the *south side* of the pole so that at some such point as X the resultant field is zero. X is termed a **neutral point,** and when the compass needle is placed there, it will show no tendency to set in any particular direction. Fig. 10 shows the type of diagram which should be obtained, and it should be observed that as the distance from the pole increases so do the lines of force approach more and more closely to those due to the earth's horizontal field alone.

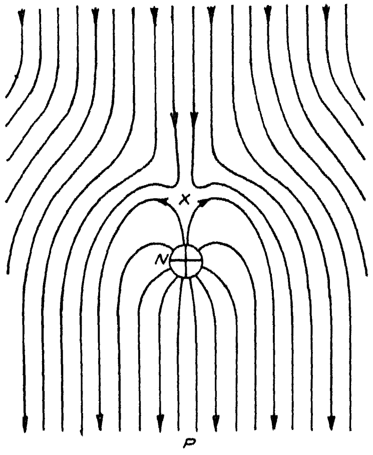


Fig. 10.

MAGNETIC SCREENING.

Experiment.—Place a bar-magnet and a soft iron ring as indicated in Fig. 11, and plot the lines of force in their neighbourhood. It will be found that the lines of force near the ring are bent towards it and tend to crowd into the iron, in fact the iron can be regarded as being a better conductor of the lines of force than air. In other words the iron is said to possess a greater **permeability** (see also pages 76, 96) than air, just as sandy soil is more permeable than clay to the passage of water.

Furthermore, in the above experiment it should be observed that inside the ring the compass needle shows no preference to set itself in any particular direction, i.e. the ring acts as a *screen* from the influence of the magnet. This fact is utilised in certain galvanometers (page 179), in wattmeters, and also in screening radio-transformers which possess iron cores. It should be realised that a ring does not provide perfect screening, but if made of soft iron the screening is more complete than if steel (which is less permeable) is used. In the use of commercial measuring instruments such as voltmeters and ammeters, it is obviously very important that the readings should be as far as possible independent of stray magnetic fields. Consequently a certain tolerance is laid down by the specification of the British Engineering Standards, which limits the variation of the maximum indication of the instrument to 3 per cent., when it is exposed to a magnetic field of 10 c.g.s. acting in the direction producing a maximum effect on the instrument readings. The effect is more pronounced with moving iron than with moving coil instruments, and is kept to a minimum value by the use of iron cases.

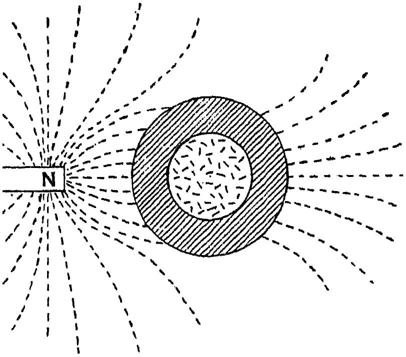
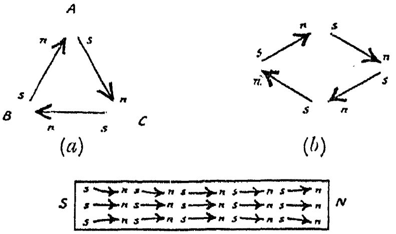


FIG. 11.

Molecular Theory of Magnetism.—The nature of the change that occurs in a rod of iron or steel when it is magnetised can be illustrated as follows:

Experiment.—Fill an empty test-tube loosely with steel filings and suspend it in a stirrup. It will be found that it comes to rest in a particular direction determined by the torsion in the suspending fibre. Next remove the tube from the stirrup and draw the pole of a bar-magnet over it several times in the same direction. It will be seen that the upper layers of the filings arrange themselves so as to point in the same way. If the tube is now carefully replaced in the stirrup it will come to rest in the magnetic meridian. Furthermore, on bringing a compass needle near the ends of the tube, it will be found that one end exhibits north polarity while the other shows south polarity. On shaking the tube, however, all signs of polarity will be found to disappear.



(a) (b)

(c)

Fig. 12.

According to the molecular theory of magnetism, which was suggested by Weber and developed by the English scientist, Ewing, the molecules in a rod of iron or steel are themselves small magnets even though the rod itself is not magnetised. In the unmagnetised state of the substance the molecules are arranged in groups or chains such as those shown in Figs. 12 (*a*) and (*b*). It is evident by considering, for example, the points A, B and C of the first group, that the close proximity of opposite and equal poles will mean that their separate effects at external points will neutralise one another, i.e. there will be no external magnetic field. The process of magnetisation breaks up these chains and causes the molecular magnets to rotate so that their axes tend to point in the direction of the magnetising force. The result is that north and south polarities appear at the ends of the rod, while the poles of molecules in its central parts are neutralised by the opposite poles of the adjacent molecules. The arrangement of the molecules in the magnetised rod is shown in Fig. 12 (*c*) and indicates when the rod is *saturated*, i.e. the body is in a condition when its magnetisation is the maximum possible.

The above description is only an outline of the molecular theory of magnetism, but it suffices to account in a general way for a large number of magnetic phenomena, such as :

1. The effect of mechanical vibration in destroying the magnetism of bodies by causing the molecules to form closed chains again. This was illustrated in the above experiment.

2. It is impossible to produce an isolated magnetic pole, for on breaking up a magnetised needle (a glass hard needle is best for the purpose) a corresponding number of *separate* magnets are always produced. Furthermore, the north and south poles are always of equal strength.

3. The change in length of a body upon magnetisation, known as the phenomenon of *magnetostriction*, vaguely indicates some molecular rearrangement. The effect is complicated, however, as some materials show an increase while others indicate a decrease in length.

4. The heating of a magnet tends to destroy its magnetism. This effect can be explained by reference to the dynamical theory of heat which postulates that the movements of the molecules of a substance become faster as the temperature increases. Consequently at high temperature it is assumed that the vibrations of the molecules become too violent for the magnetising force to be able to align them.

5. That substances cannot be magnetised above a certain value, i.e. the saturation value, has already been explained by the theory.

An interesting example of the molecular nature of magnetism is given by the alloys discovered by *Heusler* in 1903. The constituents of these alloys are copper, manganese and aluminium which in themselves are only very feebly magnetised, but when present in certain proportions they are as strongly magnetic as steel.

Also it should be noted that the heat treatment of a steel affects its magnetic as well as its mechanical properties, for a steel which has been *suddenly chilled* from a red heat has a smaller permeability in weak magnetic fields than the same composition steel which has *cooled slowly*.

The molecular theory as outlined above does not explain the initial magnetism of the molecule. Modern thought attributes this magnetism to the motion of electrons (page 23) in their orbits around the nuclei of the atoms which make up the molecules. In this theory it is the electronic orbits (these can be compared to electric currents circulating in circular conductors) which suffer rotation in an applied magnetic field. In the case of certain atoms the direction of the alignment is *opposed* to that of the field. The theory thus offers an explanation of the initial magnetism of the molecule and the unlike magnetic properties of different substances.

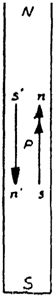
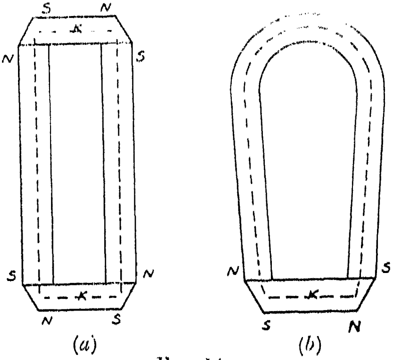


Fig. 13.

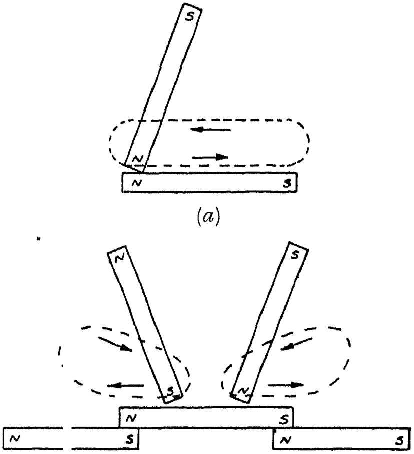
MAGNETIC KEEPERS.—As a result of the inductive action of magnetic poles, it is clear that the poles at the end of a magnet will tend to reverse the polarities of the molecules within it. For consider a point P (Fig. 13) within the body of the magnet and let *sn* represent a magnetic molecule at that point. Now the inductive action of the poles N and S of the magnet on a piece of unmagnetised material at P would be to magnetise it in the direction s^1n^1. In other words the magnet tends to *demagnetise* itself. This defect is remedied by providing magnets with **keepers,** which consist of strips of soft iron placed in positions (K) as indicated in Figs. 14 (*a*) and (*b*). The free poles at the ends of the magnets induce unlike poles on the parts of the keepers which are in contact with them. The keepers thus become magnetised, so that, while they are in position, layers of molecules are arranged throughout in continuous lines (i.e. closed circuits) with unlike poles adjacent. In this manner the presence of free polarity is avoided and the possibility of demagnetisation almost completely eliminated.



(*a*) (*b*)

Fig. 14.

METHODS OF MAGNETISATION.—An unmagnetised bar of iron or steel can be magnetised by stroking it several times in the same direction with a piece of magnetite or with a pole of a permanent magnet. The bar is then turned over and the procedure repeated, and it should be noted that the *end of the bar last touched acquires opposite polarity to that of the stroking pole* (N in Fig. 15 (*a*)). A modification of the above *method of single touch* is the *method of double* (*or divided*) *touch* in which two magnets are simultaneously used to stroke the bar. The magnetism in the bar is enhanced if its ends are allowed to rest on the poles of two other magnets during the process, as shown in Fig. 15 (*b*). It should be observed that the bar is placed so that the poles supporting it are opposite to the polarities which are to be developed on the adjacent ends of the bar.



(*a*)

(*b*)

Fig. 15.

Reference has already been made (page 1) to the *electrical method* of magnetising a bar of iron or steel. Insulated wire is wound round the specimen (Figs. 16 (*a*) and (*b*)), which is to be magnetised, and a current from a battery is passed through the coil. The polarity developed in the specimen is dependent upon the direction of current flow and is given by the following rule :

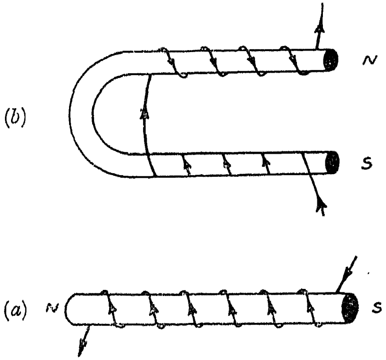


Fig. 16.

If an observer looks at one end of the coil along the axis and the current is flowing in an anti-clockwise direction through

the coil, then the pole at that end is a north pole. If the current flows in a clockwise direction the pole is a south pole.

Figs. 17 (*a*) and (*b*) furnish a useful mnemonic for remembering this rule.

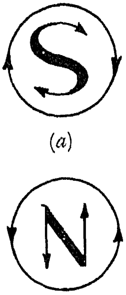


Fig. 17.

In the case of the horseshoe magnet (Fig. 16 (*b*)) the wire has to be wound in opposite directions on the two limbs. The degree of magnetisation produced will increase with the current (i) and with the number of turns (n) in the coil, until the specimen is saturated. The product $n \times i$ is known as the number of *ampere turns*. As previously mentioned (page 4), if the specimen is of soft iron and the current is stopped the residual magnetism is very feeble. This low retentivity of soft iron is frequently an advantage in certain types of apparatus, as in the case of the electro-magnet in its various types and applications. A typical laboratory electro-magnet for the production of strong magnetic fields is shown in Fig. 2, the pole-pieces P and the cores C (i.e. the portions of the magnetic circuit surrounded by the magnetising coils) being made of Swedish soft iron. With a small separation, say 1 or 2 mm., of the conical-shaped pole-pieces, magnetic fields of the order of 30,000 gauss (page 18) may be realised in the gap. Another type of electro-magnet used largely in industry for lifting purposes depends upon the attractive force of the poles for ferrous material. The attractive force between the poles and keeper of an electro-magnet can be shown to be equal to $\frac{B^2A}{8\pi}$ dynes, where B is the induction in the iron (page 95) and A is the surface area of the poles. The ordinary electric bell is another illustration of the application of electro-magnets. On depressing the bell push at P (Fig. 18) a current from a battery e is passed through the windings of a small electro-magnet E so that the iron core becomes magnetised. Consequently the soft iron armature A is attracted towards the electro-magnet. Now attached to this armature is a steel spring striker S which carries a hammer H at its end, and this latter strikes the bell B. But the movement of the armature breaks the circuit at the point of the adjustable screw, C, and so the current ceases to flow and in turn the core of the electro-magnet loses most of its magnetism. Hence the armature is no longer attracted and returns to its original position, so that the electric current again flows in the completed circuit. The movements previously described are then repeated and the ringing of the bell is thus maintained.

The Inverse Square Law.—Hitherto the properties of the magnet have been dealt with from a qualitative standpoint and no reference has been made to the magnitude of the magnetic forces. It has been stated, for example, that a north pole repels a similar pole but attracts a south pole. A little consideration will suggest that this force of repulsion or attraction is likely to depend upon the distance between the poles. This suggestion, in fact, is found to be the case, and the law connecting the force between two poles and the distance between them is known as the **Inverse Square Law.**

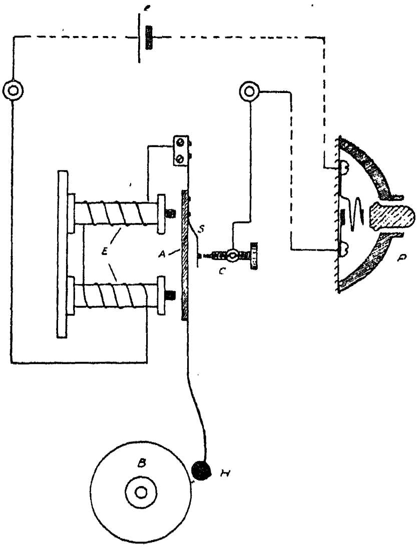


Fig. 18.

The law was established as a result of very careful experimental measurement by Coulomb, a French engineer, who lived in the latter part of the eighteenth century. The law is stated as follows:

The force between two isolated magnetic poles varies inversely as the square of the distance between them.

i.e. F (Force) $\propto \frac{1}{r^2}$ where r is the distance between poles.

It follows that if the distance between two poles is doubled then the force between them is only $\frac{1}{2^2} = \frac{1}{4}$ of its former value. This force will also depend on the strength of the poles. Thus, if the strength of one pole is doubled, it is found that the force between them is doubled, so that the law now becomes $F \propto \frac{mm_1}{r^2}$ where m and m_1 are the strengths of the two poles.

It should be carefully noted that the above law is only strictly applicable to *point poles*, and, furthermore, these are supposed to be isolated from the influence of any other magnetic forces. The magnetism of a point pole is considered to be concentrated only at a point, and the nearest approach to its realisation in actual practice is the Robison ball-ended magnet. When this magnet is carefully magnetised the poles can be considered to be concentrated at the centres of the two spheres.

Effect of Medium between the Poles.—So far in defining our law of force no mention has been made of the possibility of the force varying with the nature of the medium between the poles. Experience has shown that the complete law of force should be written as $F = \frac{mm^1}{\mu r^2}$ where μ is known as the *permeability* of the substance and will be defined later in the book (page 96). Now the value of the permeability of air differs so little from unity (the value of μ for a vacuum) that the law of force for two poles situated in vacuum (or air) may be expressed as $F = \frac{mm_1}{r^2}$. In this equation if F is given in dynes and r in cms., then m and m_1 are expressed as the number of unit poles in the poles under consideration. The definition of the unit pole follows, as, if the poles are chosen equal in magnitude so that when $r = 1$ cm., F is equal to 1 dyne, then our relation becomes $1 = \frac{m^2}{1}$ (since $m = m_1$) i.e. $m = 1$.

Hence **a unit magnetic pole is that pole which, when placed at a distance of 1 cm. in vacuo (or air) from an equal and similar pole, repels it with a force of 1 dyne.**

The unit pole is often designated the **weber.**

A Further Note on the Inverse Square Law.—The Inverse Square Law is not restricted to the case of the force between magnetic poles but is a law of very wide application. Sir Isaac Newton found that the gravitational force between two bodies varies inversely as the square of the distance between them. It will be seen in the next chapter that the law also holds for the force between charges of electricity. Again, if we consider the illumination produced on a screen by a source of light, the intensity of illumination varies inversely as the square of the distance of the screen from the source. The geometric aspect of the law can perhaps be illustrated best by considering the last case. Take a wire pyramid as represented in Fig. 19. If O is the source of light and A, B and C are removable surfaces situated at distances 1, 2 and 3 units respectively from O, then the light which falls on A is distributed over B when A is removed. The area of B is clearly four times that of A so that the intensity of illumination on B must be one-fourth of that on A.

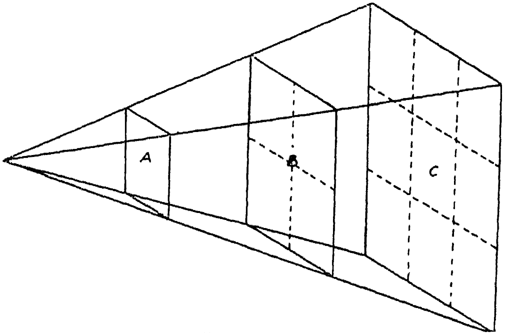


FIG. 19.

Likewise the intensity of illumination on C which is three units away from O, is one-ninth the intensity on A, and, in general, it can be said that the intensity of illumination on a surface due to a source of light varies inversely as the square of the distance of the surface from the source. In comparing with the case of magnetism, the lines of magnetic force can be considered as analogous to light rays (see para. on Alternative Definition of Field Strength).

MAGNETIC INTENSITY OR STRENGTH OF MAGNETIC FIELD.—It is universally agreed that *the intensity of a magnetic field at any point is measured* **by** *the force in dynes exerted on a unit north pole placed at that point.*

The *direction of the field* is assumed to *coincide* with that of the *force acting* on the unit *north pole*, and, furthermore, it is supposed that the presence of the latter does not appreciably disturb the original magnetic field.

A magnetic field of unit intensity is that field which exerts a force of 1 dyne on a unit pole placed in it. This **unit** of field strength is called the **gauss,** and is named after the German mathematician Carl Gauss, who did pioneer work of great importance in the theory of magnetism. It follows from the above definition that in a magnetic field of strength 5 gauss, a unit pole would experience a force of 5 dynes. The definition may also be conveniently expressed symbolically as

$$\mathbf{F=m\times H}$$

where F is the force in dynes exerted on a pole of strength m units when placed in a field of strength H gauss. If the magnetic field is due to a number of scattered magnetic poles then the resultant field strength at any point is obtained by calculating the force on a unit north pole due to each separately, and then compounding the forces by the parallelogram or polygon of forces.

The following calculations illustrate the foregoing discussions :

Example 1.—Two north poles of strengths 16 and 25 are placed at a distance of 20 cms. from each other. Find the force experienced by a south pole of strength 10 units when it is placed between them, and on the line joining them, at a distance of 5 cms. from the stronger pole.

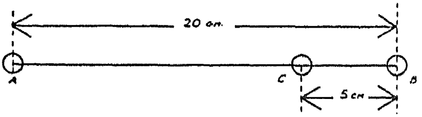


FIG. 20.

Let the pole of strength 16 be placed at A (Fig. 20) and the pole of strength 25 at B. The pole of strength 10 is at C, 5 cms. distant from B. Then attractive force (in dirn. **CA**) on the south pole at C due to A is

$$=H.m=\frac{m_1}{d^2}\times m$$

$$=\frac{16}{15^2}\times 10=\frac{32}{45}\text{ dyne.}$$

Attractive force (in dirn. **CB**) on the south pole at C due to B is $=\frac{25}{5^2}\times 10=10$ dynes.

∴The resultant force on the south pole at C in the dirn. **CB** $=\left(10-\frac{32}{45}\right)=$**9.29 dynes.**

Example 2.—A bar-magnet is 20 cms. long and its pole strength is 40 units. Find the force that would be experienced by a north pole of strength 10 units, when placed at a distance of 30 cms. from the centre of the magnet on its axis produced.

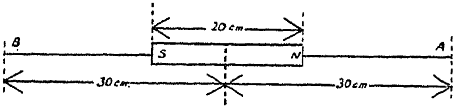


FIG. 21.

Let SN represent the bar-magnet and suppose that the given north pole is placed at A (Fig. 21), i.e. at a point distant 20 cms. from the north pole of the magnet on its axis produced.

Then Repulsive Force on north pole at A due to north pole of magnet $=\frac{40}{20^2}\times 10=1$ dyne

and Attractive Force on north pole at A due to south pole of magnet $=\frac{40}{40^2}\times 10=0.25$ dyne.

∴Resultant Force of Repulsion on north pole at A due to bar-magnet $=1.0-0.25=0.75$ dyne.

If the given north pole is placed at B, i.e. at a point distant 20 cms. from the south pole of the bar-magnet on its axis produced, the resultant force is again 0.75 dyne, but it is now one of attraction.

ALTERNATIVE DEFINITION OF FIELD STRENGTH.—The conception of a magnetic field in terms of lines of force has been formulated on a previous page and from the diagrams formed with iron filings, it has been evident that in a general way the lines are more crowded near the poles, i.e. at places of large magnetic intensity. From this aspect, therefore, the *strength of a magnetic field at any point* may be alternatively *defined by the number of lines of force passing through each sq. cm. of a plane held perpendicular to the direction of the field at that point.* Unit field is then said to have one line of force passing through one sq. cm. of area placed at right angles to the direction of the field, and a field of strength H will have H lines of force passing through the same area.

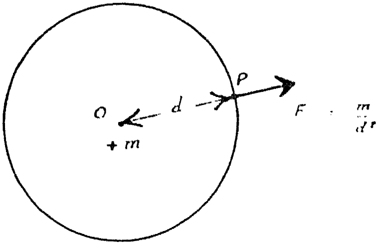


FIG. 22.

It follows from the above definition of unit field strength that 4π lines of force must be assumed to radiate from a unit pole. For consider a pole of strength m units at O (Fig. 22). Then the strength of the magnetic field at P as given by the first definition is m/d^2. But according to the alternative definition $\frac{m}{d^2}$ must represent the number of lines of force per sq. cm. passing through the spherical surface at P, since lines of force are radial from an isolated pole and hence the surface of a sphere with the pole as centre will be always normal to the direction of the lines of force.

$\therefore$ the total number of lines of force passing through this spherical surface $= 4\pi d^2 \times \frac{m}{d^2} = 4\pi m$.

i.e. $4\pi m \div m = 4\pi$ **lines of force radiate from a unit pole.**

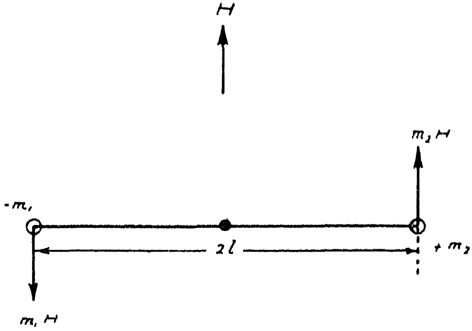


FIG. 23.

TO SHOW THE EQUALITY OF THE POLES OF A MAGNET.

Experiment.—Fix a magnetic needle horizontally to a cork which is floated in water contained in a large bowl (to avoid surface-tension effects). Remove any iron from the neighbourhood and move the cork to different places near the centre of the dish. In all cases it will be noted that there is no tendency for any translationary motion of the cork (and needle) but the cork always rotates until the needle lies along the magnetic meridian.

If m_1 and m_2 are the strengths of the north and south poles respectively of the needle (Fig. 23), then the resultant force tending to move the needle in the direction of the magnetic meridian is $m_1H - m_2H$ where H is the **uniform** horizontal intensity of the earth's field. But the above experiment has shown that there is no tendency for the needle to move in the direction of the field, i.e. the resultant force is zero.

Hence $(m_1 - m_2)H = 0$, or $m_1 = m_2$,
i.e. the poles of a magnet are equal in magnitude (although of opposite sense or sign). ✓

QUESTIONS.

1. Describe in detail two different methods of magnetising a piece of iron. Illustrate your answer by sketches, indicating clearly the North pole of the iron.

How would you prove that the iron had been magnetised? (U.L.C.I., A, 1935.)

2. Sketch the distribution of the magnetic field due to a horseshoe magnet: (*a*) without keeper, (*b*) with a keeper placed so that there is a small gap between the keeper and each pole, (*c*) fitted with pole pieces and a soft iron cylinder between the poles as in a moving-coil instrument. In each case show the polarity of the magnet and the direction of the magnetic field. (U.E.I., S1, 1935.)

3. State the Inverse Square Law as applied to magnetic poles. What is meant by unit magnetic pole, and intensity of a magnetic field at a point?

If a unit magnetic pole were placed half-way between two unlike poles of strengths 200 and 500 units respectively, which are 40 cms. apart, what would be the resultant force on it? (U.E.I., S2, 1933.)

4. Describe a method of magnetising a length of steel wire by means of four bar-magnets, showing clearly how the poles of the magnets are arranged. What is meant by consequent poles?

How would a bar-magnet with consequent poles, freely suspended on its centre, set itself with regard to the earth's magnetic field? (C. & G., 1930.)

5. What materials should be used for the magnet cores of an electric bell? In what way would the action of the bell be affected if the cores were made (*a*) of mild steel, (*b*) of hard steel, (*c*) of brass? What would you do if the armature tended to stick to the poles? (C. & G., 1913.)

6. Show by means of a sketch a bar of iron wound with some insulated wire. Show also in what direction the current must be flowing in the winding in order to make the right-hand end a north-seeking pole.

CHAPTER II

ELECTRIC CHARGES

Preliminary.—The existence of electricity in nature is manifested by such effects as lightning discharges, but generally speaking its presence is not commonly apparent, apart from examples artificially created by man. Actually the existence of electrification is usually inferred from the behaviour of material things when subjected to electrical influences, such as a shock imparted to the human body, or the decomposition of certain liquids. There is a close parallel to this experience in the case of heat, for a body can be recognised as being hot when it feels unpleasant to a finger touch.

It was typical of early scientific explanations to attribute such phenomena to the agency of fluids, and corresponding to the " caloric " fluid theory of heat, there were the early fluid theories of electricity. In time the " caloric " theory was superseded by the dynamical theory in which heat is regarded as a consequence of the energy of vibration of the molecules or ultimate particles of a body. Similarly the modern theory of electricity attributes electrification to a disturbance of the smallest particles of a body or **electrons** (derived from the Greek word for amber) as they are called, and it is now generally regarded that electricity is the fundamental stuff of the universe. Although the theories concerning the nature of matter and electricity have changed very rapidly during the last decade, it is still recognised that there are two kinds of electricity, a fact which can be shown by the following experiment.

Experiment.—Rub a rod of ebonite with catskin and show that the *part* of the rod which has been rubbed has acquired the property of *attracting* light bodies such as sawdust, bits of paper, etc. The rod of ebonite is now said to be electrically charged.

CHAPTER II

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Next suspend a charged ebonite rod in a stirrup (Fig. 24), and bring another charged ebonite rod up towards it. (*Note:* Both rods must be charged in the same manner, e.g. by rubbing with fur.) The suspended rod will suffer *repulsion.* Now charge a glass rod by rubbing it with silk and bring it up towards the suspended (and charged) ebonite rod, attraction will ensue. If, however, this charged glass rod is moved towards a suspended glass rod which has been charged in a similar manner, then repulsion occurs. Such experiments as the above led to the deduction that at least two kinds of electrification existed, and that bodies with a similar electrification repel one another, but oppositely electrified bodies are attracted to each other. Hence the fundamental law of electrostatics:

Like charges repel, unlike charges attract.

Experiment.—Bring a charged ebonite rod towards a small pith ball hanging by a dry silk suspension from a stand, an arrangement known as a pith ball electroscope. The pith ball will be initially attracted to the rod, but after contact will suffer repulsion. This effect may be explained by presuming that the pith ball *on contact* with the rod acquires the same *kind* of electricity as the latter, so that repulsion occurs as a consequence of the above law.

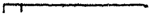


FIG. 24.

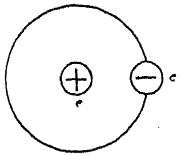
In order to distinguish between the different kinds of electrification the terms positive (+) and negative (−) are used. It is conventional to regard an ebonite rod rubbed with fur as being −vely electrified, whereas a glass rod rubbed with silk is said to have a charge of +ve electricity. The notation, however, is purely arbitrary, and in the light of modern experience it would have been better if the chosen nomenclature had been reversed.

It should be noted also that the sign of electrification produced by "rubbing" or "friction" depends on the type of rubber, as well as on the material of the rod, for example a glass rod when rubbed with fur becomes −*vely* electrified. Following the discovery of the existence of two kinds of electricity, *Du Fay*, a French soldier of the early eighteenth century, advanced the theory that there were *two kinds of electric fluid*, and that in a neutral body these fluids were present in equal amounts, but they could be separated by friction. On this theory the state of electrification of a body depended upon the particular fluid which was in excess. *Benjamin Franklin* (1706-1790) later attempted to simplify the two-fluid theory and propounded a *one-fluid theory*, and a modification of this theory has been largely used as a means of explaining many well-known facts of experiment. In this theory a body is said to be +vely or −vely electrified according as to whether it possesses respectively an excess or deficit of this weightless fluid, or " electric fire " to use Franklin's phrase.

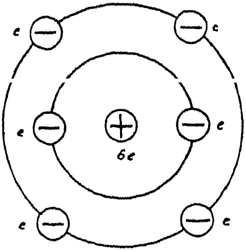
The modern electron theory shows a similarity to Franklin's theory except that now a *+vely charged body* is *due to* a *deficit of* the *−vely charged electrons*, while a −ve charge results from a surplus of electrons. The electron theory received its inception at the instance of Sir Ernest Rutherford's experiments on radioactivity and was subsequently developed by other scientists. The main development is generally attributed to Niels Bohr, the present Professor of Physics at Copenhagen, who was a research student under Rutherford at Manchester University. According to this startling and beautiful theory the atom of any substance consists of a *heavy +vely charged* core or **nucleus** round which a number of −ve charges (electrons) travel at high speeds, like so many planets round the sun. The resultant charge on the nucleus is +ve, as despite the possible presence of electrons on the core, the number of protons (i.e. positive units of electricity) present will always be in excess. Furthermore, it follows that the resultant +ve charge on the nucleus must be equal in magnitude to the total −ve charges of the revolving electrons, since in normal circumstances the atom as a whole is neutral, i.e. shows no charge.

The number of electrons surrounding the nucleus varies from one in the case of hydrogen (Fig. 25) to 92 for uranium, and the intervening kinds of atom can be consecutively arranged such that any particular atom has one more and one less electron respectively than the preceding and succeeding atoms. The number of revolving electrons in an atom of an element is called the **atomic number** of the element, a term introduced by a brilliant English scientist named

Moseley, who was unfortunately killed in the Great War. The order of this list of atomic numbers follows that of the atomic weights of known elements and of those which are suspected to exist. Since nearly all the mass of an atom is concentrated in the nucleus, the atomic weights of different elements are almost completely dependent upon the difference in the masses of their nuclei.



HYDROGEN



CARBON

FIG. 25.

All electrons have the same mass which is only 1/1850th of the mass of the nucleus of the lightest atom, that of hydrogen. The mass of an atom of hydrogen is $1{\cdot}64 \times 10^{-24}$ gm.

The actual volume occupied by the nucleus of an atom and its electrons is extremely small in relation to the space in which they move, in fact the relative volumes bear a similar ratio to those of a speck of dust and a large hall. The minuteness of an electron or nucleus is further emphasised when the smallness of the atom itself is realised, e.g. there are over 10^{18} atoms in a drop of water.

A large number of electrical phenomena may be explained in terms of the electron theory, and as an example the mechanism of the charging of an ebonite rod by friction will be reviewed in the light of this theory.

In some substances the outer electrons of the atoms are more firmly bound than in others, hence it is conceivable that by bringing these into intimate contact, e.g. by rubbing two particular substances together, the outer electrons of a small fraction of the atoms of one substance will migrate to the adjoining atoms of the other substance. In the case of the ebonite rod and fur, the electrons lost by the fur are gained by the ebonite which thus becomes —vely charged, and it is evident that the fur must show a +ve charge equal in magnitude to the sum total of that of the lost electrons. This deduction can be verified by experiment, using a gold leaf electroscope. It has been estimated that with the strongest possible electrified ebonite rod only about 1 in 10^{12} of the total electrons are transferred.

CONDUCTORS AND INSULATORS.—It was known to the Greeks over 2,000 years ago that amber acquired the property of attracting light bodies after it had been rubbed, but it was not until the latter half of the sixteenth century that Dr. Gilbert, who will be mentioned again in connection with magnetism, discovered that a large number of substances possessed this property. In the case of some substances the charge remains on the part where it is developed (cf. ebonite rod in experiment on page 23), while in the case of others it distributes itself all over the body and, where possible, finds a channel through which it passes to earth or other connected bodies.

Substances along, or through, which electric charges do not readily travel are known as *insulators* (or non-conductors), and typical examples are ebonite, amber, shellac, sulphur, glass, porcelain and silk. On the other hand substances which offer a comparatively easy path for the passage of electric charges are known as *conductors*. The best conductors are the metals, followed by carbon and salt solutions.

It should be noted that the distinction between the two classes is really a matter of degree only, as it is impossible to obtain a perfect conductor or a perfect insulator. Again with some materials it is difficult to place them definitely in either class, for a substance such as dry wood is a partial conductor when used with high voltages (i.e. high electrical pressures), but can be employed as an insulator in low voltage circuits.

Experiment.—Hold a rod of brass in the hand and rub it with a catskin. Bring the rubbed end near some small pieces of paper. No evidence of electrification will be apparent. Next mount the brass rod in an ebonite handle, and, taking hold of the latter with one hand, rub the brass as before. It will now be found that the brass rod attracts pieces of paper when it is brought near them.

In explanation of the above experiment, it is evident that in the first case the charge developed on the brass rod was conducted through the rod itself, and thence through the human body to *earth*. In the second case, however, the insulating ebonite handle proved a barrier through which the electricity could not readily pass.

Consequently *to discharge an electrified conductor* it is only necessary to *earth it* by touching with a finger (assuming no possible danger of shock, and providing the observer is not insulated from earth by possessing rubber soles to his boots), or by a metal wire connection to a water pipe.

THE GOLD-LEAF ELECTROSCOPE.—This is a very convenient instrument for detecting the presence of electric charges and exists in a number of different modifications. A very simple form of the instrument is shown in Fig. 26, and consists of a metal support B, to the bottom end of which two gold leaves *gg* are attached by a thin coating of shellac, care being taken that metallic contact between leaves and support is preserved. The leaves and part of the support are contained in a glass bell jar J which is fitted with a stopper E made of sulphur or similar high insulating material. The rod B which carries a metal cap C, passes centrally through the insulating stopper and is supported by it.

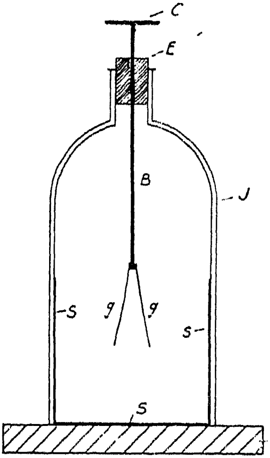


FIG. 26.

SS are strips of tin or lead foil fixed to the walls of the case opposite the gold leaves, and also to the wooden base W of the instrument. When the gold leaves are charged, the *induced charges* (page 30) on the strips SS tend to further increase the deflection, i.e. make the instrument more sensitive. If the deflection becomes very large the leaves will touch the strips and become discharged, thus preventing the possible fracture which might occur through excessive divergence.

Another type of electroscope is shown in Fig. 27, and in this form the container J is a metal box provided with glass windows for viewing the gold leaf *g*. Only one leaf is used, and when undeflected it rests on the flat brass plate fixed to the lower end of the rod B. The sensitivity of this instrument may be increased by moving inwards the metal plate P, which is in metallic connection with the case J. The latter can be effectively connected to "earth" by joining a thin wire to the terminal T, the other end of the wire being soldered to a water or gas pipe.

If the gold leaf and rod B are required to hold a charge for any appreciable time, it is very necessary that the air within the vessel J should be kept dry. Hence a suitable receptacle R (Fig. 27), containing a drying agent such as phosphorus pentoxide or silica gel, should be placed inside the instrument.

In order to charge an electroscope by contact, rub an insulated brass conductor with fur and bring it into contact with the metal cap C of the instrument. The leaves diverge and remain so on removing the conductor, and the electroscope is said to be —vely charged. Discharge the leaves by earthing the cap. Repeat the experiment with the brass conductor using india-rubber instead of fur as the rubber, and so charge the electroscope +vely.

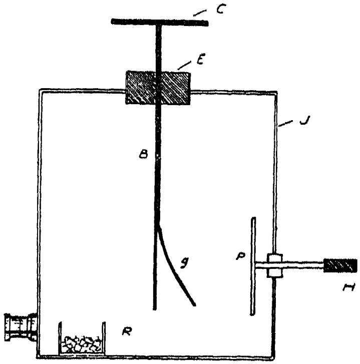


FIG. 27.

Experiment.—To test whether a body is electrified or not, and if so what sign of electrification it possesses, three electroscopes will be required. They must be respectively +vely charged, —vely charged and neutral, or alternatively one electroscope may be utilised to assume each state in turn.

If no divergence of the leaves is produced on bringing up the body to the cap of the neutral electroscope, then the body is uncharged as either a +vely or —vely charged body will produce a deflection of the leaves.

On slowly bringing up a +vely charged body to the +vely charged electroscope an increased divergence will be obtained, but a —vely charged body will cause the leaves to collapse. If, however, the body has a large —ve charge and is brought very close to the cap the leaves may open out again. This effect is due to electrostatic induction and is comparable to the magnetic case already discussed on page 3. Hence it is very necessary to move a body *slowly* towards the electroscope and observe the initial effect on the leaves.

A +-vely charged body on *slowly* approaching the −vely charged electroscope will cause the leaves to collapse, and it should be noted that the same effect is produced by bringing up an earth-connected conductor to the instrument.

Hence the *only sure test for the sign of electrification* of a body *is* that if an *increased divergence* of the leaves is obtained on *slowly* bringing up the body to the cap of the electroscope, then the body is electrified similarly to the electroscope, i.e. either +ve or −ve.

A PROOF PLANE consists of a copper (or brass) disc or ball, say about ½ in. to 1 in. diameter, mounted at the end of an ebonite rod (Fig. 28) as handle. When the disc is placed in contact with the surface of a conductor it becomes a portion of this surface and will acquire a charge dependent upon the amount of charge on the conductor at the point of contact. By removing the proof plane in a direction normal to the surface, the sign and magnitude of the acquired charge may be detected and measured by an electroscope. It should be noted that the fraction of the total charge of the conductor which is removed by the proof plane is small, if its area is kept small relative to the total area of the conductor.

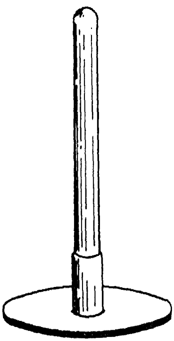


FIG. 28.

INDUCED ELECTRIC CHARGES.—The fact that bodies can be electrically charged by induction was discovered by Canton, an English schoolmaster, who lived in the eighteenth century. The following experiment illustrates *a method of charging a body by induction :*

Experiment.—Bring a −vely charged ebonite rod near one end of a narrow sausage-shaped brass conductor which is mounted on an insulating stand (Fig. 29). Next, while the ebonite rod is held in position, touch the end X of the conductor with a proof plane and show that the latter is −vely charged by producing an *increased* divergence of a −vely charged electroscope when the instrument is approached. In a similar manner show that the end Y has a +ve charge, in this instance using a +vely charged electroscope for its detection.

On removing the charged rod the conductor will be found to be discharged, from which fact it is deduced that the +ve and −ve charges at Y and X respectively, were equal in magnitude.

Repeat the experiment, but, with the charged ebonite rod in position, earth the conductor by finger touch. Withdraw the ebonite rod and test for electrification on the conductor by means of a proof plane. It will be found to be −vely charged.

If possible a modification of the above experiment should be performed in which two equal insulated metal spheres X and Y, initially uncharged and in contact, are separated while under the influence of the charged ebonite rod, and the sign of their charges examined.

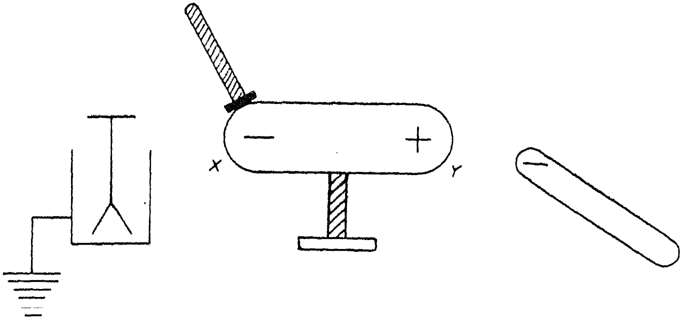


FIG. 29.

In explanation of the above experiment it is said that the −ve electricity on the rod repels the free electrons of the brass conductor to the further end X, leaving a deficiency at Y, which means that this nearer end has become +vely electrified. When the conductor is earthed by momentarily touching with a finger the electrons at X travel to earth through the human body. On removing the ebonite rod the +ve charge which was previously *bound* becomes *free*, and distributes itself over the whole of the conductor, which has thus been charged by **induction.**

It should be noted that when a +vely charged conductor is earthed, electrons are said to pass from the earth to the conductor until the charge is *neutralised*.

Experiment.—The easiest and best way of charging an electroscope is by the method of induction. In order to charge an electroscope —vely, for example, obtain a +vely charged rod and gradually move it towards the electroscope until the required divergence of the leaves is obtained. With the rod held in this position (Fig. 30 (*a*)), earth the electroscope by touching the cap with the finger (Fig. 30 (*b*)). The leaves will collapse. Now insulate the cap and the leaves by removing the finger, and note that the leaves still remain closed (Fig. 30 (*c*)), but that upon the removal of the rod they open out to their previous extent and the electroscope has been —vely charged by induction (Fig. 30 (*d*)). The student should explain the results of this experiment in a similar manner to the explanation given for experiment on page 31.

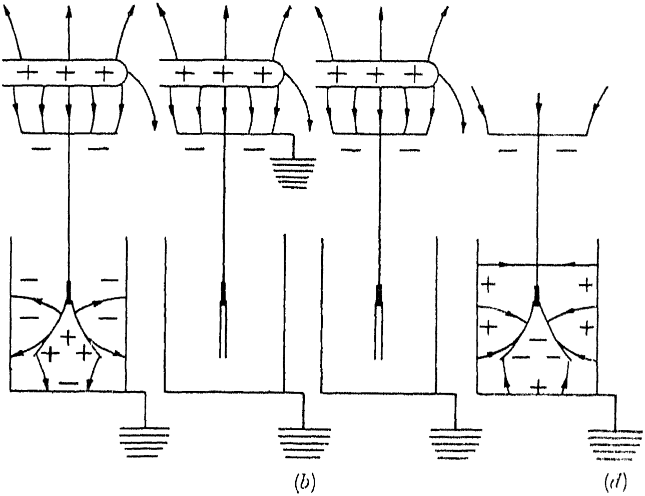
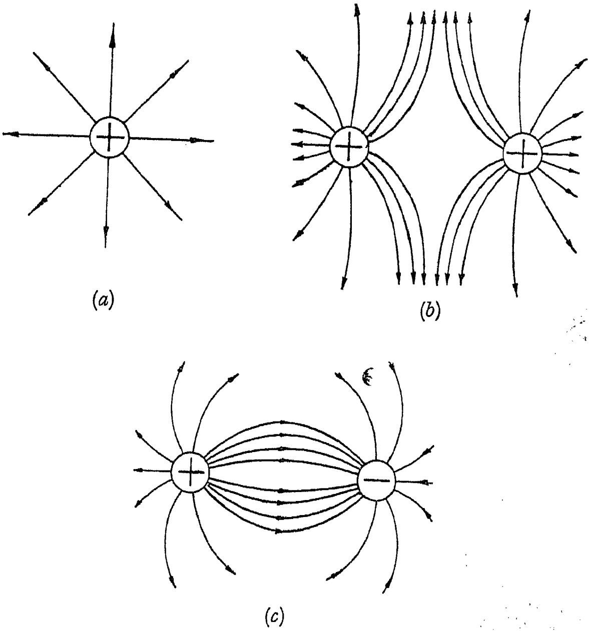


FIG. 30.

ELECTRIC LINES OF FORCE.—In a similar manner to the corresponding magnetised body, an electric charge is regarded as subjecting the surrounding space (or ether) to a state of strain which can be represented by so-called electric lines of force. These latter have similar properties to magnetic lines of force, viz. (1) they are subject to a tension along their length, (2) they repel each other laterally, and (3) they do not intersect; furthermore, the +ve direction of a line of force is assumed to be the direction in which a free +ve charge would move. Fig. 31 shows the distribution of lines of force in the neighbourhood of (*a*) a charged spherical conductor, (*b*) and (*c*) two equal spherical conductors having equal charges respectively similar and dissimilar.



(*a*) (*b*)

(*c*)

FIG. 31.

The electric fields in the above cases may be mapped out in a manner analogous to the use of iron filings, by means of oxalic acid crystals, or by a mixture of crushed gypsum and carborundum crystals scattered on a horizontal glass plate. On the underside of the plate are affixed two discs of metal foil cut to the desired shape, and each is connected to a pole of an electrical machine (page 60) or high tension (i.e. high voltage) generating set. Under the influence of the electric field the crystals become *polarised* and tend to set themselves with their longest axes in the direction of the field.

The ELECTRIC FIELD of an electrified body is the surrounding space in which the influence of the charge may be detected. An interesting example illustrating the application of the idea of lines of force to a particular problem is afforded by the charging of an electroscope by induction (see Figs. 30 (*a*), (*b*), (*c*) and (*d*)).

It should be carefully noted by reference to the above Fig. 30 that the lines of force commence on a + ve charge and end on a —ve charge, and that they end always on the surface of a conductor and do not pass through it. Again it is also evident that the lines of force from the leaves end on the *earth-connected box* which thus acts as *an electric screen* from electric forces outside.

LAW OF FORCE BETWEEN ELECTRIC CHARGES.—The law of attraction and repulsion between electric charges is similar to the corresponding law in the case of magnetic poles. It is another example of the inverse square law and is as follows: *The mechanical force between two electric charges varies inversely as the square of the distance between them.*

FIG. 32.

The law was first verified experimentally by Coulomb, using his torsion balance, which consisted (Fig. 32) of a long light insulating rod carrying a small fixed gilt pith ball at each end. The rod was suspended by a fine wire to rest horizontally inside a glass case. One of the balls was charged, so that when a similarly charged pith ball *f* was placed near it, repulsion ensued and was balanced by the *torsion* of the wire.

The non-existence of lines of electrical force inside a charged hollow sphere as shown by experiment, is now taken as the generally accepted verification of the law, as Cavendish showed mathematically that this state of affairs would only exist if the inverse square law was true.

Expressed symbolically if e and e_1 are the magnitudes of the two charges in electrostatic units and r cms. is the distance between them, the force F between the charges when situated in air (or more strictly vacuo) is given by

$$F=\frac{ee_1}{r^2}\text{ dynes.}$$

If $F=1$ dyne, $r=1$ cm. and $e_1=e$, then it immediately follows that $e=1$, i.e.,

The electrostatic unit of charge (or quantity) is that charge which when placed 1 cm. in vacuo (or air) from an equal and similar charge, repels it with a force of 1 dyne.

It is important to notice that the above law requires modification when media other than air (or vacuo) separate the charges. If paraffin wax replaces air as the intervening medium then the force is reduced to $\frac{1}{2 \cdot 3}$ of its previous value, while in the case of water the expression for the force is

$$F=\frac{1}{81.}\frac{ee_1}{r^2}.$$

In general the force between two charges is given by

$$\mathbf{F}=\frac{\mathbf{1}}{\mathbf{K}}\cdot\frac{\mathbf{ee_1}}{\mathbf{r^2}}$$

where K is a constant depending upon the medium separating the charges. This constant will be seen later to be an important factor in the theory of condensers, and it is known as the *specific inductive capacity* (S.I.C.) or *dielectric constant* of the medium. A table of values of K for a number of common substances is given in the Appendix.

It should be mentioned here that air (or any other gas) is normally a very poor conductor of electricity owing to the firmness with which the electrons are held by the molecules of the gas. Under certain circumstances, however, electrons may be detached temporarily from some of the molecules and in turn they may attach themselves to other molecules. Consequently +vely charged and —vely charged molecules will exist in the gas ; these are known as **ions**, and as such they are free to move independently under the action of an electric field. The gas is now said to be "*ionised,*" i.e. a conductor of electricity.

This condition may be brought about in a number of different ways, one method consists in creating an intense electric field. Hence the few ions which are always present in a gas are accelerated to such velocities, that the violence of their collisions with neutral gas molecules cause the latter to be ionised (see paragraph on action of points, page 40).

Example 1.—Two equal conducting spheres are insulated and charged with −25 and +15 units of electricity respectively. What is the force between the spheres when the distance between their centres is 20 cms.? If the two spheres are now momentarily placed in contact and are then removed to their previous positions, how have the magnitude and nature of their mutual force changed?

Before the spheres are brought into contact the force between them $= \frac{q_1 \cdot q_2}{r^2}$ dynes $= \frac{(-25) \times (+15)}{20^2} = -\frac{15}{16}$ dyne,

i.e. a force of *attraction* (—ve sign) of $\frac{15}{16}$ dyne.

On bringing the two spheres into contact the total charge, i.e. $-25 + 15 = -10$ units will spread itself equally over the spheres, since they are identical in size, and each will receive $-\frac{10}{2} = -5$ units. Hence the force between the spheres *after contact* will be $\frac{(-5) \times (-5)}{20^2} = +\frac{1}{16}$ dyne.

i.e. the force has been reduced in magnitude and is now *repulsive* (+ve sign).

Example 2.—Small spheres carrying charges +2, −4, +6 and +8 units are placed in order at the angular points of a square ABCD of side 12 cms. Determine the magnitude and direction of the force on a charge of −3 units, which is placed at the point of intersection of the diagonals.

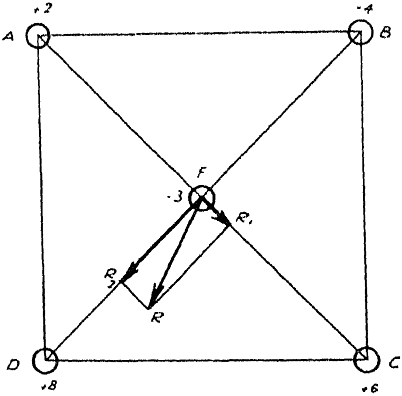


FIG. 33.

The charges are distributed as in Fig. 33. The charge at the intersection of the diagonals is at a distance of $6\sqrt{2}$ cms. from each of the other charges.

Force on charge -3 units at F due to charge $+2$ units at A

$$= \frac{2 \times -3}{36 \times 2} = -\frac{1}{12} \text{ dyne (}-\text{ve sign indicates attraction).}$$

Force on charge -3 units at F due to charge $+6$ units at C

$$= \frac{6 \times -3}{36 \times 2} = -\frac{1}{4} \text{ dyne.}$$

$$\therefore \text{ Resultant } R_1 = \frac{1}{4} - \frac{1}{12} = \frac{1}{6} \text{ dyne in direction FC.}$$

Force on charge -3 units at F due to charge -4 units at B

$$= \frac{-4 \times -3}{36 \times 2} = +\frac{1}{6} \text{ dyne.}$$

Force on charge -3 units at F due to charge $+8$ units at D

$$= \frac{8 \times -3}{36 \times 2} = -\frac{1}{3} \text{ dyne.}$$

$$\therefore \text{ Resultant } R_2 = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \text{ dyne in direction FD.}$$

Hence the magnitude of the resultant force on the charge -3 at F is R, where

$$R = \sqrt{R_1^2 + R_2^2} \quad \text{(since } R_1 \text{ and } R_2 \text{ are at right angles)}$$

$$= \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{10}}{6} \text{ dyne} = \mathbf{0.53 \text{ dyne.}}$$

To determine the direction of R it is easily seen that

$$\tan \angle \text{RFC} = \frac{R_2}{R_1} = \frac{\frac{1}{2}}{\frac{1}{6}} = 3$$

whence $\angle$ RFC (from trigonometrical tables) $= 71° 34'$. But $\angle CGF = 180° - \angle GFC - \angle FCG = 180° - 71° 34' - 45° = 63° 26'$ i.e. the resultant force makes an angle of 63° 26′ with the line joining DC. (G is point on DC cut by FR).

INTENSITY OF ELECTRIC FIELD.—In a similar manner to the case of a magnetic field, an electric field is regarded as the space surrounding an electrified body (or system of charges) where its influence can be detected. Further, an

electric field is said to have unit intensity at any point when it exerts a force of 1 dyne on a unit pole placed there, and the direction of the field is given by that in which a unit +**ve** charge moves. It is tacitly assumed that the introduction of the charge does not disturb the original field.

It should be carefully noted that if a charge of 2 c.g.s. units is acted on by a force of 10 dynes when placed in an electric field, the strength of the field at that point is $\frac{10}{2}=$ 5 c.g.s. units and **not** 5 dynes.

Example.—Calculate the strength of an electric field at a point distant 10 cms. in air from the centre of a conducting sphere carrying a charge of 50 c.g.s. units.

The charge of 50 units may be considered as acting at the centre of the sphere, hence the force on unit charge at a point distant 10 cms. away will be given by $F=\frac{q_1 q_2}{r^2}$, where $q_1=$ 50 c.g.s., $q_2=1$ c.g.s. and $r=10$ cms.

$$\text{i.e. } F=\frac{50}{10^2}=\tfrac{1}{2} \text{ dyne.}$$

∴ The strength or intensity of the field at a point 10 cms. away from the centre of the sphere is 0.5 c.g.s. unit.

DISTRIBUTION OF ELECTRICITY ON CHARGED CONDUCTORS.—In order to determine the amount of charge per unit area or *surface density* of electricity at any point on a conductor, an area surrounding the point is taken and is chosen small enough so that the distribution of charge over it may be assumed uniform. The surface density is then equal to the total charge on this chosen surface divided by its area, i.e. it is equal to the quantity or charge of electricity per unit area round the point.

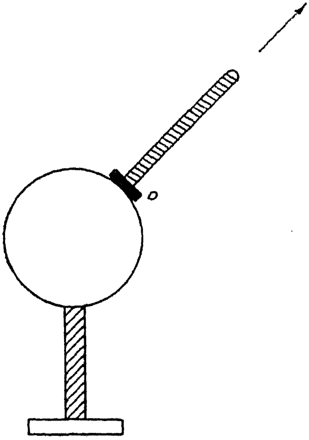


FIG. 34.

The fact that an electric charge distributes itself uniformly over a conductor of regular shape, i.e. the sphere, may be experimentally verified by means of an electroscope fitted with a graduated scale placed behind the gold leaves.

Experiment.—Charge an insulated brass sphere and place the disc (D) of a proof plane in contact with a portion of its surface (Fig. 34). Remove the disc in a direction normal to the surface and transfer the acquired charge to an electroscope. Note the divergence of the leaves. Repeat the procedure, when it should be noted that the divergence is the same whichever part of the surface of the charged sphere is touched.

Replace the sphere with an irregularly shaped conductor and note that the distribution of charge is not uniform.

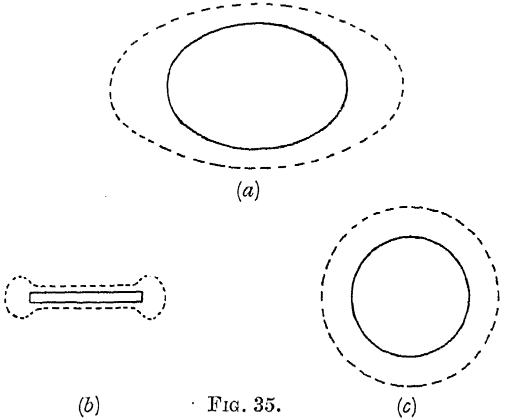


Fig. 35.

Fig. 35 shows the density of charge over differently shaped conductors, the separation between the dotted line and the outline of the conductor at any point being a measure of this quantity. In the case of an ellipsoid (Fig. 35 (*a*)), the density has its maximum value at the ends of the longest diameter and is a minimum at the ends of the shortest, while a flat disc (Fig. 35 (*b*)), shows a high concentration of charge at the edges. The general rule for any *isolated solid conductor* is that the *surface density* is uniform over flat surfaces but *increases as the curvature becomes greater.* (Note that the curvature at any point is given by $\frac{1}{r}$, where r is the radius of the sphere which closely fits the contour of the surface of the conductor at that point

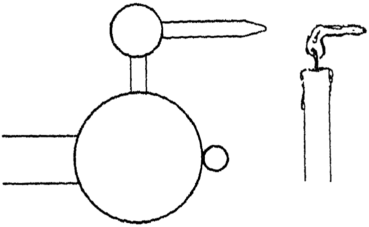


Fig. 36.

This tendency of electricity to collect at sharp edges and points explains the well-known phenomenon of the *discharging action of sharp points.* The high density of charge at a point attracts the dust and air particles in its neighbourhood, which on contact with the conductor become similarly charged. They are then repelled and carry away the acquired charges with them. The stream of particles driven away from the point in this manner constitutes an *electric wind.* This wind can be maintained by connecting the point to the stationary conductor of a Wimshurst electrical machine (page 60), and may be sufficiently strong to blow out a candle flame (Fig. 36).

Newton's Third Law of Motion tells us that action and reaction are equal and opposite, and the action of the electric windmill (or electric whirl) is due to the fact that when the electrified particles near a point are repelled, there must be a corresponding reaction on the point. The windmill consists of a light cross of wire (Fig. 37) with the ends sharpened and bent at right angles, so that they all point in the same direction. When this cross is pivoted on a needle point and charged by an electrical machine a stream of charged particles issues from each point, and the reactions produced at these points cause a rotation of the mill in an *opposite* direction to that in which the points are bent.

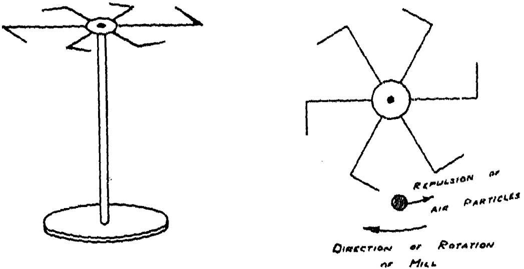


Fig. 37.

An important application of the theory of the action of points is in the use of *lightning conductors* to protect buildings from damage due to lightning discharges. A lightning conductor consists essentially of a broad strip of iron or copper (alternatively thick stranded wires) which is carefully insulated from the walls of the building. The conductor terminates at its upper end in a number of sharp points, and is efficiently *earthed* in moist soil at its lower extremity. The presence of a +vely charged cloud above the conductor (Fig. 38) induces −ve electricity at its upper extremity, while the induced +ve charge is neutralised by the passage of electrons from earth. Furthermore, since the upper end is pointed an intense electric field will be produced there so that the air becomes ionised, and the cloud will thus tend to become discharged by the stream of −ve ions attracted towards it.

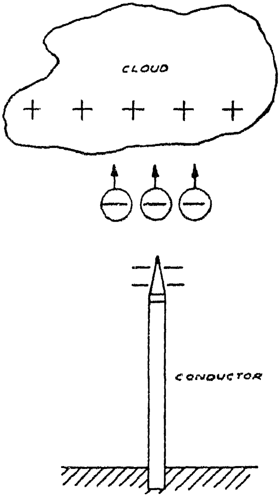


FIG. 38.

The discharging action of points will be mentioned again later in connection with the Wimshurst Machine (page 60).

It should be mentioned here that when the air surrounding a conductor becomes strongly ionised the discharge is known as a " corona " discharge, and may be visible as a pale violet light. When the electrostatic stress is increased its place is taken by a " brush " discharge (consisting of pale blue light " streamers " diverging outwards from the conductor in the form of a brush and accompanied by a hissing sound) until finally there is the " flash-over " to an adjacent conductor, i.e. the spark discharge with its distinctive " crack."

In high voltage overhead transmission lines a " corona " discharge may take place from the conductors, and also in cables where air films exist. The effect is also liable to occur in transformers, switchgear, etc., and any possible trouble due to this cause is reduced to a minimum by the employment of rounded parts and earthed metallic screens.

It is evident that if conductors are to retain their charges they must not possess sharp edges or points, and furthermore electrical apparatus should be kept free from dust particles, as the latter act as points and tend to lower the effective insulation, i.e. to dissipate the charge.

QUESTIONS.

1. Distinguish between conductors and insulators and give examples of each. Explain why some bodies readily conduct electricity while others do not.

2. Describe an experiment to show that both positive and negative charges are produced in equal quantities by frictional methods. How is this fact explained by theory ?

3. How does the shape of a charged body affect the distribution of charge on its surface ?

Illustrate your answer by showing the distribution of charge on the following isolated conductors, a sphere, a cone, a cube and a circular disc. Explain the action of a sharp point on a highly charged conductor. (C. & G., 1934.)

4. Explain briefly what is meant by an "induced electric charge." Describe an experiment to show that the total induced charges on a body are equal to the inducing charge. (C. & G., 1933.)

5. A small insulated sphere A which is charged with 20 units of positive electricity attracts a similar insulated sphere B which is charged with 12 units of negative electricity with a force of 60 dynes. The two spheres are allowed to touch each other momentarily and are then replaced in their original positions.

What is the force now exerted between the two spheres ? Is it attraction or repulsion ? What will be the force exerted if the distance between the spheres is halved ? (C. & G., 1928.)

6. Give sketches showing (*a*) the electrostatic lines of force due to an insulated negatively charged ball, and (*b*) the distribution of the lines when a similar ball, also charged negatively, is placed some distance from the former. How would the force between the balls be altered by doubling the distance between them ? (C. & G., 1920.)

7. Describe the construction of a gold leaf electroscope. How may it be used to determine whether a body is electrified or not, and if so what sign of electrification it possesses ?

CHAPTER III

FURTHER ELECTROSTATICS

ELECTRIC POTENTIAL.—The idea of electric potential was first suggested by the Hon. Henry Cavendish (1731-1810), who was one of the most extraordinary characters in the history of science. Cavendish did not actually use the term potential but wrote of the "*degree of electrification*" of a body.

Experiment.—Give a negative charge to the ebonite disc E (Fig. 39) of an **electrophorus** by rubbing it with catskin, and holding the insulating handle H, place the metal cap C on the charged disc. The surfaces of the disc and cap are sufficiently irregular that actual contact between them is made only at a few points. The metal cap consequently is charged by induction, and on touching it the induced negative charge passes to earth. The cap may now be removed with its charge of +ve electricity, and the least distance is noted at which a spark will not pass between it and a knuckle of a finger presented near to one edge of the electrophorus. If the experiment is repeated after a larger charge has been given to the ebonite disc, then a spark will pass at a *longer* distance than before.

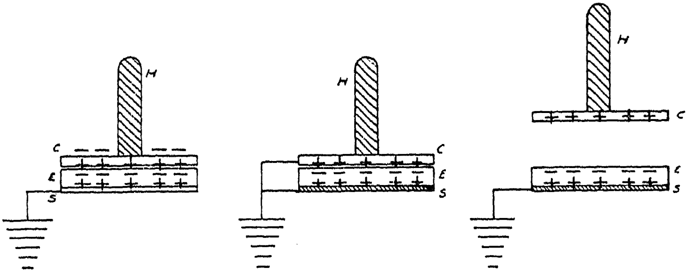


FIG. 39.

In the above experiment it is the difference between the "degrees of electrification" (or potentials) of the cap and the knuckle which is the cause of the spark.

It is convenient to regard the earth as being at zero potential so that all earth-connected conductors are at zero potential. In the earliest development of electrical theory attention was always focussed on the supposed movement of +ve electricity. Hence, since the cap in the above experiment is at a +ve potential (i.e. higher potential than the earth), when it is earth-connected +ve electricity will flow from the conductor to earth. Adopting a modern perspective, this transition constitutes a flow of electrons from the earth to the conductor. It should be noted that a −vely charged isolated conductor is considered to be at a −ve potential (i.e. a potential below that of the earth). Hence if two conductors, charged to different potentials, are placed in electrical contact, there will be a flow of +ve electricity from the one at the higher to the one at the lower potential, until both conductors have attained the same potential. It follows that all points on a charged *conductor* must be at the same potential.

Fig. 40.

Electrical potential is analogous to *level* in hydrostatics, and to *temperature* in heat. In the latter case the temperature of a body is defined as its "degree of hotness," and it is the condition which determines whether heat will flow from the body to other bodies, or vice versa. Likewise *the electric potential of a body is the condition which determines whether electricity will flow from the body to the earth or vice versa.* It will be useful to consider the hydrostatic analogy more fully by referring to Fig. 40, where A and B are two vessels of relative sizes as indicated. On opening the tap T, water will flow from B to A, despite the fact that the quantity of liquid in A is greater than that in B, i.e. it is the *pressure head* of water which *determines the direction of flow of the liquid,* and this flow will be maintained until the level of the liquid in the two vertical tubes is the same. Electric potential is analogous to this hydrostatic pressure and is referred to often as electric pressure. In this connection it should be remembered that Faraday considered the medium surrounding a charged body to be in a state of strain, and that insulators such as air are able to withstand this electric pressure, or potential, in varying degrees, but that when a conductor forms a part of the surrounding medium, a movement of electrons in the conductor results in the phenomenon of induction.

Now consider an isolated +vely charged conductor, and suppose that a unit +ve charge is brought up from an infinite distance towards the conductor. At any point the unit charge will be under a force of repulsion which will vary inversely as the square of its distance from the conductor. Hence work will be performed against this force of repulsion, and the expended energy will be stored up as potential energy (i.e. "energy of position") in the charged body; for, if free to move, the unit charge would be repelled to an infinite distance by the charge on the conductor. Consequently any point in the space surrounding a charged body may be considered to possess a potential due to the presence of the charge, and in fact the *difference of potential* between two points X_1 and X_2 can be measured by the amount of work done in moving a charge from X_1 to X_2.

The above conception of potential leads to a definition of the unit of potential, viz. **if one erg of work is performed in bringing a unit charge from one point to another, the difference of potential between these points is one electrostatic unit (E.S.U.).**

It should be noted that the difference of potential is only *numerically* equal to the amount of work done on unit charge, and is *not* expressed in ergs (see example below).

Note should be made of the fact that if a body is −vely charged it attracts the unit +ve charge referred to, and *does* work in drawing the charge to itself. The energy of the system thus decreases and the potential of the body is said to be negative.

From a mathematical point of view the potential due to a charged body at an infinite distance will be zero, as no force will be exerted on a unit +ve charge placed there, consequently the *potential of a charged body* will be measured by (i.e. is *numerically* equal to) the work done (in ergs) in bringing a unit +ve charge from infinity up to the body.

In actual practice it is found that a more convenient unit is the *volt* (named after the scientist Volta) which *is* 1/300*th of the E.S.U. of potential.*

The student should now review afresh the question of induction from the point of view of potential. As an example consider the case represented by Fig. 29, page 31. It is evident that before the introduction of the sausage-shaped conductor, the potential at Y due to the −vely charged rod will be less than that at X, as the potential will *rise* from the −ve value of the ebonite rod to zero at the earth-connected walls of the room. In other words there will be an electric pressure acting in the direction XY. It follows that when the conductor is inserted in position, this difference of potential will cause a supposed flow of +ve electricity from X to Y in order that the potential of the conductor should become uniform. In the light of modern experience this means that there is a flow of electrons from Y to X, so that X becomes −vely charged and Y +vely charged.

It can be shown that the potential at a point situated (in air) a cms. from a charge $+q$ is equal to $\frac{q}{a}$, but if the space between the charge and the point A is occupied by a medium of specific inductive capacity K, the potential at A due to the charge is $\frac{1}{K} \cdot \frac{q}{a}$.

The electric potential at a point due to a number of charges, is obtained by adding together the potentials due to the separate charges, due regard being attached to their signs.

Example.—Charges of 8, −12, and 3 electrostatic units respectively are placed at the angular points A, B and C of a square ABCD of side 6 cms. Find the potential at the point D (Fig. 41).

Potential at D due to charge $+8$ at $A = +\frac{8}{AD} = +\frac{8}{6}$

" " " " " " -12 at $B = -\frac{12}{BD} = -\frac{12}{6\sqrt{2}}$

" " " " " " $+3$ at $C = \frac{3}{CD} = +\frac{3}{6}$

$$\text{Total Potential at D} = +\frac{8}{6} - \frac{12}{6\sqrt{2}} + \frac{3}{6}$$
$$= 0.42 \text{ e.s.u. approx.}$$

FURTHER EXPERIMENTS WITH AN ELECTROSCOPE.

Experiment.—Place the electroscope on a slab of paraffin wax to insulate it from earth, and charge the cap and leaves —vely say, as in the experiment on page 32. The leaves will diverge.

Connect the cap and the metallic case of the instrument by using a length of wire fixed to an insulating holder, e.g. an ebonite rod. The leaves will collapse. The cap (including leaf system) and case now form one conductor, and electrons will flow from the former to the latter until their potentials are equalised. Hence although the electroscope is still charged the leaves are undeflected, as no potential difference exists between the leaves and the case.

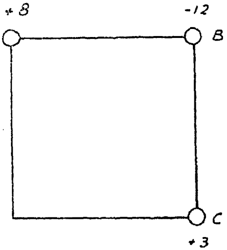


FIG. 41.

Now earth the case, and the leaves will immediately diverge, since there is now a potential difference between the leaves and the case, the latter being at zero potential. It should be noted that a similar divergence would be observed if the cap, and not the case, is earthed.

It follows from the above experiments that if an electroscope be provided with a scale behind the leaves, the instrument may be utilised for the approximate determination of potential differences. In such measurements the case is usually maintained at zero potential by connection to earth. Instruments used for the actual measurement of differences of potential are known as *electrometers*.

The student should perform the following experiment to test the sensitivity of the electroscope shown in Fig. 42.

Experiment.—Connect the cap C (Fig. 42) of the electroscope to the +ve terminal of an H.T. (High Tension) battery (200 volts) and join the —ve terminal via a high resistance R (e.g. an electric lamp) to the terminal T. Observe the deflection on a scale placed behind the leaf, or by means of a microscope provided with an eye-piece scale). Take a number of readings of deflection and voltage by varying the tapping point on the H.T. battery. Plot a curve connecting deflection with applied voltage. The curve will obviously represent a calibration of the instrument for a given position of the plate P.

Interchange the battery connections and show that the deflection is unaltered. In consequence of this result the electroscope may be used for alternating current (A.C.) measurements (page 266). Verify this deduction by substituting the A.C. supply mains (220 volts) for the H.T. battery and note the deflection.

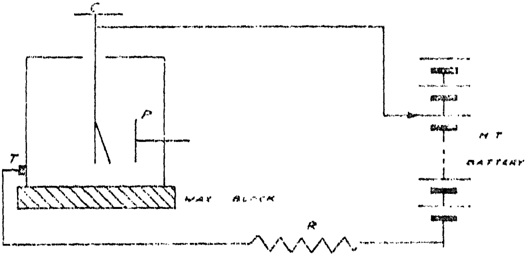


FIG. 42.

Experiment.—To verify experimentally that all points on the surface of a charged solid conductor are at the same potential (a result which has been previously anticipated), proceed as follows:

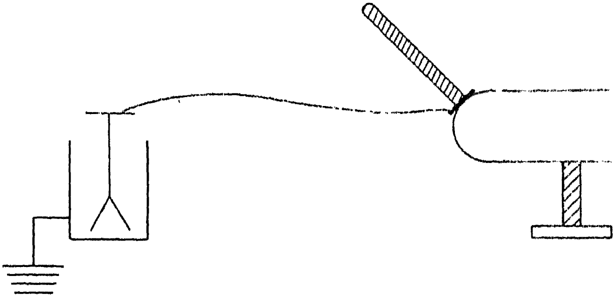


FIG. 43.

Connect a thin copper wire from the cap of an uncharged electroscope to the metallic disc of a proof plane (Fig. 43). Using the ebonite handle move the proof plane to different places on the surface of the conductor, and it will be observed that the same divergence of the leaves is always obtained. This method of using the proof plane in combination with the electroscope to measure potential should be contrasted with its previous use in measuring electric charge.

Some Experiments with Hollow Conductors.

Experiment.—The purpose of this experiment is to show that the charge resides on the outside of a charged tin can, in the absence of any charged bodies inside the vessel.

Charge a tall tin can (i.e. the height should be about three times the diameter) which is insulated by a wax block. Verify that there is a charge on the outer but no charge on the inner surface, by using a proof plane and electroscope as in the experiment on page 30.

To investigate the potential at various points of the system, connect the proof plane to the cap of the electroscope by a fine copper wire soldered at each end to a small metal ball. Move the proof plane from place to place and observe that the divergence of the leaves is constant, indicating that the space well inside the can is at the same potential as the can itself.

Again since the potential in the air space is uniform no work is done in moving a charge from place to place inside the can. Hence **no electric force exists inside a hollow charged conductor provided there is no charged body inside.**

Experiment.— *Charged body inside a tin can*— Faraday's Ice Pail Experiment.

Suspend a metal ball by a long silk thread. Connect the insulated can V with the electroscope by a thin wire (Fig. 44), and let the system be initially neutral. Charge

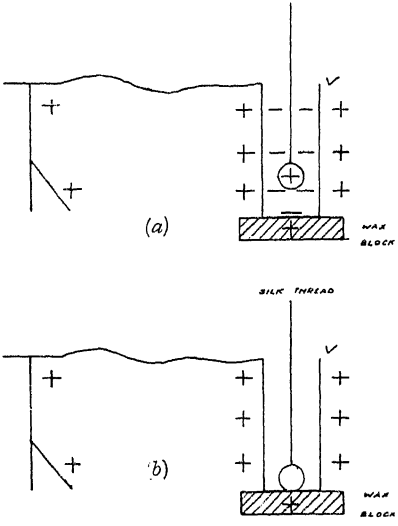


FIG. 44.

the ball +vely, say, and gradually lower it inside the metal vessel. The leaves of the electroscope will show an increased divergence until the ball is well inside the can (Fig. 44 (*a*)).

Allow the sphere to touch the can and then withdraw it. No change in the divergence of the leaves is noted, and furthermore the sphere is completely discharged (Fig. 44 (*b*)). It should be noted that on touching the can the +ve charge on the ball must exactly neutralise the −ve charge induced on the inner surface of the can, i.e. *the induced and inducing charges are equal in magnitude.* Furthermore it is evident that to completely discharge a conductor it is only necessary to allow it to touch the inner surface of a closed hollow conductor.

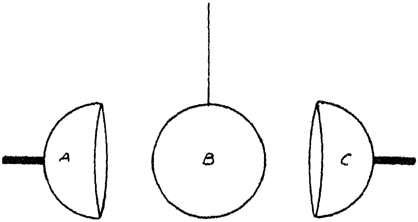


FIG. 45.

An important result arising from the above experiments is that the inside of a hollow conductor is screened from the effects of external fields. Interesting applications of this fact are the employment of *wire cages to screen electrical apparatus* required in high voltage testing stations, and the use of metallic coatings on the surfaces of wireless valves to minimise the effect of stray fields.

The above experiment has shown that an electric charge resides on the outer surface of a hollow conductor, and it may now be verified experimentally that this applies also to a solid conductor.

Experiment.—The following experiment is due to Cavendish, but is generally attributed to Biot.

A and C (Fig. 45) are two hemispherical metal caps provided with insulating handles. The two caps can be fitted exactly over a metal sphere B which is suspended by an insulating thread.

Charge the ball B −vely, say, and fit the uncharged caps over it.

The outside of the caps will now show a −ve charge. Carefully remove the caps by pulling them aside simultaneously, and verify that the sphere B is completely discharged and that the charge has passed to A and C.

In experimental work, as stated previously, the earth is taken to be at zero potential. It is, however, obvious that the earth must be continually receiving large accessions of electricity, and that it must also be frequently losing charges by various means. The **capacity** of the earth, however, is so great that these changes do not appreciably affect its potential, so that the latter can be assumed to have a constant zero value. The above problem may be likened to the hydrostatic analogy of an ocean of water, whose level (cf. potential) will be unaffected by the addition or subtraction of many tanks full of liquid (cf. quantity), since the capacity of the ocean is so large. As a consequence the heights of bodies on the earth's surface are referred to sea-level (cf. zero of electric potential). This argument suggests a point which is of considerable importance. The degree to which a body becomes electrified when a charge is imparted to it, depends not only upon the magnitude of the charge but also upon the size of the body.

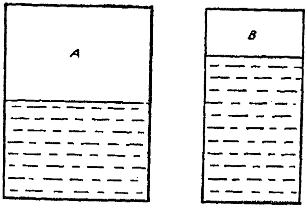


FIG. 46.

It is useful to consider, as previously (page 44), a hydrostatic analogy of the electrical problem, the hydrostatic head and the quantity of water corresponding to electrical potential and charge respectively. A and B (Fig. 46) are two vessels of the same height, but the cross section of A is greater than that of B, i.e. the volume or capacity of A is larger. If now a given volume of water is poured into each of these vessels the liquid will rise to a greater height in B than in A, which has the larger capacity. Translated into the terms of the electrical case it means that *the potential* (V) *of a body when a given charge* (*Q*) *is imparted to it is inversely proportional to the capacity* (*C*) *of the body,* i.e. $V \propto \frac{1}{C}$ when Q is constant. Again if different quantities of water are poured into any one vessel the height of the resulting liquid column is directly proportional to the total quantity poured into the container. In the electrical sense, therefore, the potential of a body is directly proportional to the charge imparted to it, i.e. $V \propto Q$ when C is constant. Combining the above relations it is evident that by suitable choice of units, $V=Q/C$ or $C=Q/V$, i.e.

$\text{Capacity} = \dfrac{\text{Charge}}{\text{Potential}}$. In words it signifies that the **capacity of a condenser is measured by the ratio of the charge (or quantity) received by it, to the potential acquired in respect of this charge.**

If $Q=1$ e.s.u. and $V=1$ e.s.u., then $C=1$ e.s.u., so that **a conductor has a capacity of one electrostatic unit if the electrostatic unit of quantity raises its potential by unity.**

The electrostatic unit of capacity is very small in comparison with the practical unit known as the Farad.
1 Farad $=9\times10^{11}$ e.s.u. and **1 Micro-farad** (μF) $=$ **10^{-6} Farad.** The micro-farad is the order of the capacity which is most frequently used, as the farad is excessively large when it is considered that the capacity of the earth itself is only approximately $\dfrac{1}{1400}$ of a farad.

Another practical unit of capacity, or **capacitance** as it is now termed, that is sometimes employed in the specification of condensers is the *jar*, one jar being equal to 1/900 μF. approximately.

It can be shown that the capacity of a sphere in electrostatic units is equal to its radius in cms.

THE CONDENSER.—It follows from the expression $C=Q/V$ that if the potential of a conductor is lowered while the charge remains constant then its capacity is increased. This result may be achieved by the introduction of an earthed plate into the neighbourhood of the conductor. Such an arrangement of conductors which is designed to increase the capacity of one of them is known as a condenser. In Fig. 47, A is an insulated +vely charged conductor whose potential is measured by the work that is necessary to carry unit +ve charge from some such point as P up to its surface. It is seen that a —ve charge will be induced on the earthed connected plate B, which is placed near A, and the complementary +ve charge goes to earth (or more strictly speaking electrons flow from the earth to neutralise the +ve charge). The work that

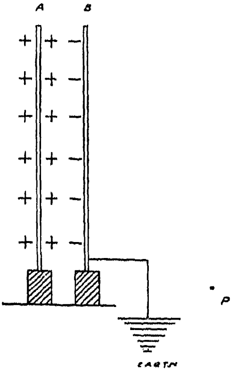


FIG. 47.

is now necessary to take unit +ve charge from the point P to the plate A is diminished owing to the " neutralising " effect of the induced −ve charge of B on the +ve charge of A. In other words the potential of A is diminished but its capacity is correspondingly increased, so that a greater charge is necessary to raise the potential of A to its former value.

The condenser shown in Fig. 47 is a form of parallel plate condenser, and if A is the area (sq. cms.) of one of the plates, and d cms. is the distance between them, then the capacity C of the condenser (with air as the insulating medium) is given by $C=A/4\pi d$ electrostatic units.

Experiment.—Set up a simple condenser as indicated in Fig. 47, using two pieces of tin plate each about 8 in. by 12 in., and two insulating blocks of paraffin wax. The plate B is connected to earth and plate A to the cap of an electroscope, the case of the latter being earth-connected. Charge the plate A and observe the change of deflection of the gold leaf, (*a*) when the plates are moved further apart, and (*b*) when a slab of wax or ebonite is inserted between the plates. Interpret your results remembering that $Q=CV$ on the usual notation, and that Q in this case is constant.

The foregoing experiment shows that the capacity of an air condenser is increased by using another dielectric such as wax or ebonite. The *ratio of the capacity of a condenser with a given dielectric to the capacity of the same condenser with air as the insulating medium,* is known as the **specific inductive capacity** (S.I.C.), or the **dielectric constant,** of the material, with respect to air.

The formula for a parallel plate condenser, in which the dielectric is a medium of specific inductive capacity K, is given by $C=\dfrac{KA}{4\pi d}$.

Example.—A parallel plate air condenser is constructed of two plates, the area of each of which is 100 sq. cms. If the distance between the plates is 0·5 cm., what is the capacity of the condenser (1) in e.s.u. and (2) in microfarads ?

What would be the capacity in microfarads if a sheet of paraffin wax of S.I.C.=2·3 occupied the space between the plates ?

$$\text{Now } C = \frac{A}{4\pi d} = \frac{100}{4\pi \times 0{\cdot}5} = 15{\cdot}9 \text{ e.s.u.}$$

$$\text{But 1 microfarad } (\mu\text{F.}) = 9 \times 10^5 \text{ e.s.u.}$$

$$\therefore\ C = \frac{15{\cdot}9}{9 \times 10^5} = 0{\cdot}000018\mu\text{F.}$$

Let C_1=capacity of the condenser with paraffin wax as dielectric.

Then $\frac{C_1}{C} = 2{\cdot}3$ or $\mathbf{C_1} = 2{\cdot}3 \times C = 2{\cdot}3 \times {\cdot}000018 = \mathbf{0{\cdot}0000414\ \mu F.}$

Choice of Dielectric.—The ability of the insulating material of a condenser to withstand high applied voltages is known as its *dielectric strength*, and is expressed in terms of the volts per cm. (or volts per mil.) which will cause the breakdown of the dielectric by sparking through it.

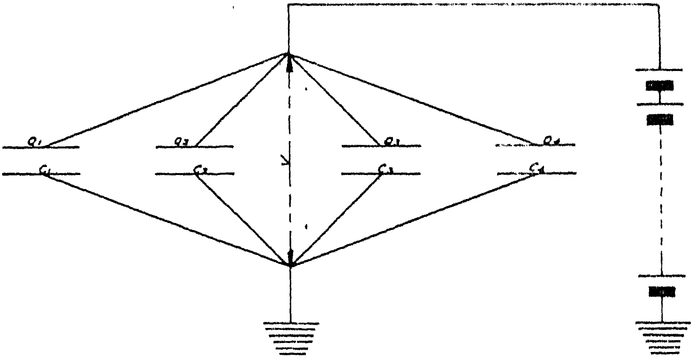


FIG. 48 (*a*).

In the case of solid dielectrics mica is superior to paraffin paper, etc., but its high cost restricts its use to the higher grade condenser and to values of capacitance of about 1 μF. Air has a lower dielectric strength and S.I.C. compared with solid and liquid insulators but is free from leakages and dielectric hysteresis.* The plates of a standard air condenser have necessarily to be separated at certain points by solid dielectrics possessing high insulation, and amberoid (formed by compressing amber shavings) is often used for this purpose. Compressed air (or carbon dioxide) is employed largely as a dielectric for high voltage work, as the dielectric strength of the gas increases with pressure, thus permitting condensers of a given capacity to be reduced in size. Oil is also used as a dielectric.

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* If an *alternating* voltage is applied to an insulating material, energy is lost due to dielectric hysteresis, which is analogous to magnetic hysteresis (page 97).

CONDENSERS CONNECTED IN PARALLEL OR IN MULTIPLE ARC.—Condensers are said to be connected in parallel or in multiple arc when their positive plates are connected to each other as are also their earthed plates.

Let Q_1, Q_2, Q_3 and Q_4 be the charges on four condensers which are connected in parallel (Fig. 48 (*a*)) and let C_1, C_2, C_3 and C_4 be the corresponding capacities. Let Q be the total charge, C the joint capacity of the four condensers, and let V be the common potential. Then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$\therefore CV = C_1V + C_2V + C_3V + C_4V$$

$$= (C_1 + C_2 + C_3 + C_4)V$$

$$\text{Hence } \mathbf{C} = \mathbf{C}_1 + \mathbf{C}_2 + \mathbf{C}_3 + \mathbf{C}_4.$$

The argument obviously holds for any number of condensers. Hence *the capacity of any number of condensers which are connected in parallel is equal to the sum of the separate capacities.*

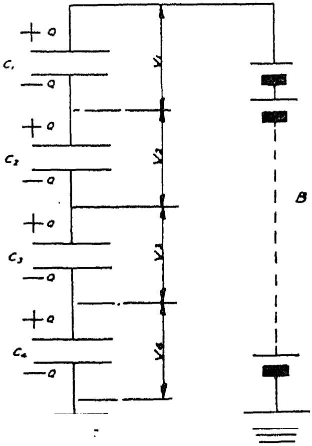


FIG. 48 (*b*).

CONDENSERS CONNECTED IN SERIES OR IN CASCADE.—Condensers are said to be connected in series when the negative plate of each is connected to the positive plate of the next and the last negative plate alone is earthed.

Let C_1, C_2, C_3 and C_4 be the capacities of four condensers which are connected in series (Fig. 48 (*b*)). If a charge $+Q$ is imparted to one plate of the condenser whose capacity is C_1, a charge $-Q$ is induced on the opposite plate. Since this is connected to a plate of the condenser whose capacity is C_2 a charge $+Q$ is developed on that plate, for, since the total initial charge on these two plates was zero, the total final charge must be zero. It follows that the four condensers must have charges $+Q$ and $-Q$ respectively on their plates. Let V_1, V_2, V_3 and V_4 respectively be the potential differences between the plates of the condensers whose capacities are C_1, C_2, C_3 and C_4. Also let V be the potential difference between the extreme plates of the series, and let C be the capacity of the four condensers joined in series.

$$\text{Then } V = V_1 + V_2 + V_3 + V_4$$

$$\text{and } \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \frac{Q}{C_4}$$

$$\text{or } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

The argument must clearly hold for any number of condensers. Hence, *when conductors are connected in series the reciprocal of the equivalent capacity is equal to the sum of the reciprocals of the separate capacities.*

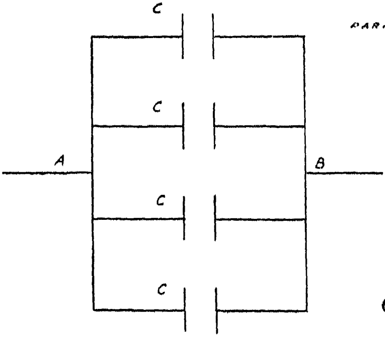


FIG. 49.

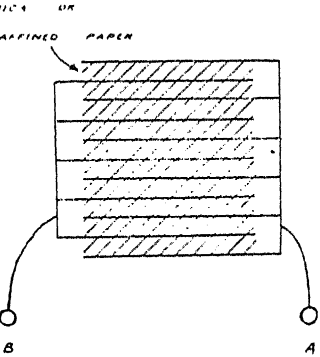


FIG. 50.

It should be carefully noted that these laws for condensers are in direct contrast to those holding for resistances (Chapter VI).

Example.—Show diagrammatically how to connect four condensers, each of 0·25 microfarad capacity, so that their combined capacity is equal to one microfarad.

The method of connecting the condensers is shown in Fig. 49. Since the capacity of each condenser is $\frac{1}{4}$ μF., the combined capacity when they are connected in parallel between A and B is $4 \times \frac{1}{4}$ μF or 1 μF.

Types of Condensers.—Two types of parallel plate condensers are constructed, viz. (1) those which have a fixed capacity, and (2) those which have a variable capacity.

A convenient type of parallel plate condenser with a fixed capacity is shown in Fig. 50. It is usually constructed of alternate sheets of tin or copper foil which are separated from each other by insulating strips of paraffined paper or mica. The odd sheets are joined together and connected to the terminal A, while the even sheets are joined to the terminal B.

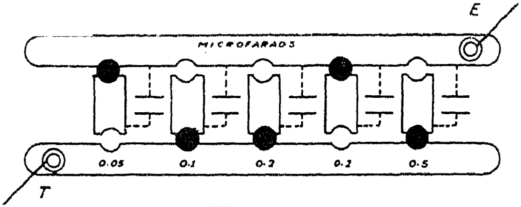


FIG. 51.

Fig. 51 gives a diagrammatic representation of one method of connection of a standard 1 μF. condenser adjustable in steps of 0·05 μF. from 0·05 to 1·05 μF. T and E are the terminals of the apparatus. The dotted lines indicate the connections to the capacities below the lid of the box, and the student should verify that with the plugs inserted as in the figure the combined capacity of the condenser (the separate condensers being in *parallel*) is $0{\cdot}1+0{\cdot}2+0{\cdot}5=0{\cdot}8$ μF.

The capacity C of a multiple plate condenser such as the one described above is given by

$$C=\frac{KAn}{4\pi d} \text{ e.s. units}$$

where K=S.I.C. of the insulating medium between the plates.
A=area of one side of a plate in sq. cms.
n=number of spaces between the plates (if N=number of plates, then n=N−1).
d=distance between two successive plates in cms.

$$\text{Also } C=\frac{KAn\times 10^{-5}}{9\times 4\pi d} \text{ μF.}$$

Example.—Find (1) in microfarads (μF) and (2) in micro-micro-farads ($\mu\mu F$) the capacity of a condenser which has 21 plates 2 in. by 1 in. if the plates are separated by mica sheets of thickness ·02 in. (S.I.C. of mica $= 6 \cdot 64$).

The capacity in micro-farads is given by the formula:

$$C = \frac{KAn \times 10^{-5}}{9 \times 4\pi d}$$

Where K = S.I.C. of medium between the plates.
A = area of one side of plate in sq. cms.
n = number of spaces between the plates.
d = distance between two successive plates in cms.

$$\therefore C = \frac{6 \cdot 64 \times (2 \times 2 \cdot 54) \times (1 \times 2 \cdot 54) \times 20 \times 7 \times 10^{-5}}{9 \times 4 \times 22 \times \cdot 02 \times 2 \cdot 54}.$$

$= \cdot 003\ \mu F$ approx.

1 microfarad $= 10^6$ micro-micro-farad ($\mu\mu F$).

Hence the capacity in micro-micro-farads is given by

$C = \cdot 003 \times 10^6\ \mu\mu F$ approx.
$= 3000\ \mu\mu F$. approx.

THE LEYDEN JAR is an early type of condenser (Fig. 52) in which the bottom and lower parts of a glass vessel are coated on the inside and outside with lead or tin foil. A brass rod which is fitted in the lid of the jar carries a knob at its top, and has a chain attached to it which makes contact with the inner metallic coating. The jar may be charged by means of a Wimshurst Machine. The tendency for leakage of charge between the metal coatings is minimised by covering the glass surface with shellac varnish.

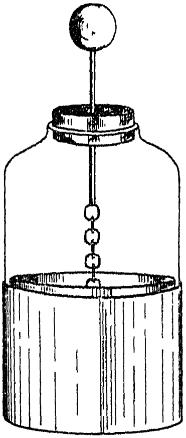
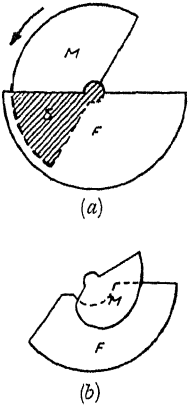


FIG. 52.

An interesting example of a cylindrical type of condenser is a power cable in which the central conductor forms the inner plate, and the surrounding conducting sheath forms the outer plate. In the case of a submarine telegraph cable the ocean itself forms the outer coating, and it is due to the time which is required for this condenser to be charged up that the speed of transmission of signals is retarded.

Variable Condensers.— The ordinary type of variable air condenser consists of a set of parallel metal plates fixed to an axle so that they can move in the air gaps between a similar set of plates fixed on an insulating framework. In Fig. 53 (*a*) is shown a movable plate M in such a position that the shaded area S is covered by the fixed plate F, the angle of rotation of the spindle being indicated by a pointer. According to the shape of the plates the capacity may be made proportional to the angular setting (Fig. 53*a*), or to the square of the angular setting (Fig. 53 (*b*)); this latter type, known as a Square Law Condenser, is used in wireless tuning circuits.



(*a*)

(*b*)

Fig. 53.

The capacity of a condenser can also be altered by varying the distance between the plates. Strips of insulating mica are placed between the plates and the pressure applied to them can be varied by means of a screw. In this way the distance between the plates can be altered in a continuous manner.

Electrolytic Condensers depend upon the formation of a complex oxide film on the surface of an aluminium plate

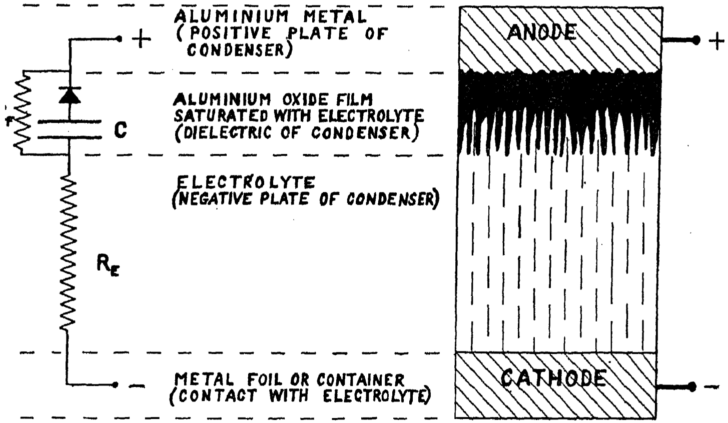

ALUMINIUM METAL (POSITIVE PLATE OF CONDENSER)
+
r
C
ALUMINIUM OXIDE FILM SATURATED WITH ELECTROLYTE (DIELECTRIC OF CONDENSER)
ELECTROLYTE (NEGATIVE PLATE OF CONDENSER)
R_E
METAL FOIL OR CONTAINER (CONTACT WITH ELECTROLYTE)
–
ANODE
+
CATHODE
–

Fig. 54.

which offers a very high resistance to the passage of a current from the film to electrolyte, but a correspondingly low resistance in the opposite direction. As the insulating film is very thin there is a large capacity existing between the aluminium plate and the electrolyte, which thus constitutes the other plate of the condenser. The obvious disadvantage of this type of condenser is its unsuitability for Alternating Current work. In Fig. 54 the equivalent electrical circuit of the condenser is also shown, the capacity at the dielectric being represented by C, the effect of power losses (see footnote page 54) in the dielectric by the resistance R_F, and the resistance of the electrolyte by R_E.

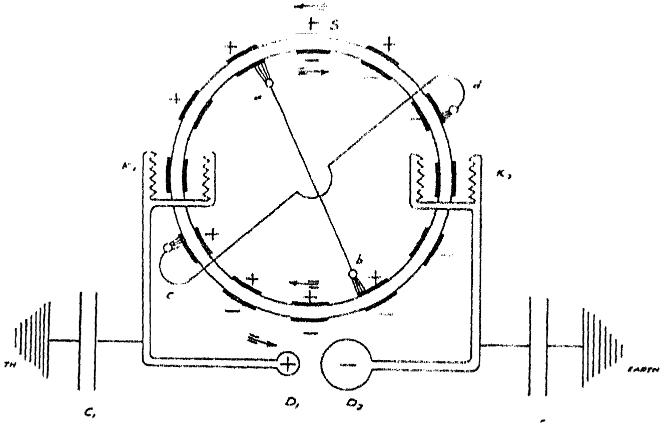


Fig. 55.

The Wimshurst Machine.—It was stated earlier in this chapter (page 58) that the Leyden Jar can be charged by means of the Wimshurst machine. This machine is named after James Wimshurst, its inventor, and the principle on which it works is similar to that of the electrophorus (page 43). The machine consists of two shellac-coated circular glass plates (Fig. 55), which are mounted close to one another on a horizontal spindle and geared so that they can be made to rotate in opposite directions by means of a handle. Narrow radial sectors of brass are fixed at equally spaced distances on the outer surfaces of the discs. When the discs revolve the sectors are lightly grazed by wire brushes which are fixed at the ends of two earthed conductors. These conductors are placed at right angles to each other on opposite sides of the plates.

The mode of action of the Wimshurst machine is simply explained by considering the plates as equivalent to two concentric cylinders rotating in opposite directions (Fig. 55). *ab* and *cd* are the earthed conductors, while K_1 and K_2 are collecting combs respectively connected to the discharging knobs D_1 and D_2. To start the action of the machine suppose a small +ve charge is given to a sector S on the outer disc. As the machine is rotated this charged sector will come opposite to a sector at *a* on the opposing disc, which is in contact with a brush connected to the cross conductor *ab*. By electrostatic induction this sector will acquire a −ve charge which it will carry away on breaking contact with the brush. When this sector comes opposite to a sector *d* on the other plate which is in contact with a brush of the cross-conductor *dc*, electrostatic induction will again take place. It is evident that the sectors in the upper half of each disc are both charged inductively, but that the two sets are of opposite sign. These charges are carried towards the collecting combs K_1 and K_2, the mode of action of the latter being previously considered on page 40 under " the action of points." Further, the charges developed on the sectors in the lower halves of the discs are such that the same kind of charges approach the same collecting comb as with the upper halves of the discs. In this way the potential difference between the knobs D_1 and D_2 will increase until a discharge takes place between them, provided, of course, that the knobs are sufficiently near to one another. In order that a larger *quantity* of electricity should accumulate before a discharge takes place, the knobs are connected to the inner coatings of Leyden Jars as represented diagrammatically by C_1 and C_2 in Fig. 55. In using the machine to charge a conductor the discharging knobs are separated and one is preferably connected to earth. The other knob is then placed in electrical contact with the conductor to be charged.

THE MEASUREMENT OF HIGH VOLTAGES.

The Spark Gap is the simplest method, and its use depends upon the empirical relation existing between any given voltage and the length of the air gap across which a spark will pass. The unknown voltage is usually applied across the

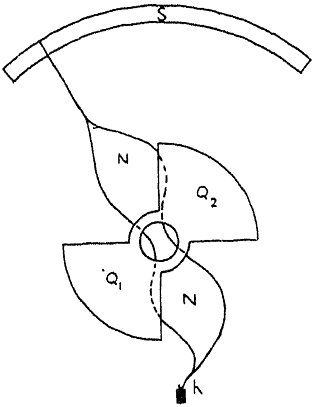


FIG. 56.

gap between two insulated spheres, and one of these is adjusted in position until a discharge passes. Voltages up to 100 kv. or more may be measured by this means. A 2-inch spark in air between 2-inch diam. polished metal balls denotes a potential difference of approximately 100,000 volts.

The Electrostatic Voltmeter is dependent upon the electrostatic attraction between two plates charged to different potentials. It consists essentially of an aluminium needle NN (Fig. 56), mounted so that it can swing freely in a vertical plane between four fixed brass quadrants, the two in front of the needle (Q_1 and Q_2) are only visible in the diagram. A small hook *h* is fixed to the lower end of the needle, and by hanging different weights on this hook, the *gravity control* may be altered to vary the range of voltages that can be measured. Since the deflection is proportional to the *square* of the applied potential difference it is independent of its sign, and the instrument may thus be used for A.C. measurements. To extend the range of the instrument above 10,000 volts, a device (Fig. 57) known as a condenser multiplier is used. E is the electrostatic voltmeter and C_1 and C_2 are two known capacities joined in series across the voltage V to be measured. It is easily verified that $V_2 = VC_1/(C_1 + C_2)$, hence V is calculable if V_2 can be measured by the voltmeter.

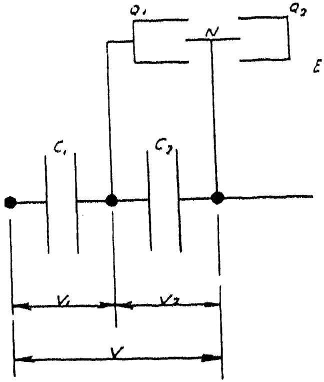


FIG. 57.

QUESTIONS.

1. Describe the electrophorus and carefully explain the principle of its action. (C. & G., 1913.)

2. An insulated cylindrical conductor is brought near to a positively charged sphere without touching it.

Describe in detail how you would utilise an electroscope to determine the condition of the cylinder as to its charge and potential. Give diagrams with your explanation.

What would be the effect of (*a*) earthing the cylinder momentarily and then (*b*) removing the cylinder from the neighbourhood of the sphere? (U.E.I., S1. 1932.)

3. Explain what is meant by the "capacity" of an insulated conductor. What does the value of the capacity of a condenser formed of two parallel plates depend upon?

Two condensers have a capacity of 4 micro-farads and 6 micro-farads respectively. What is the capacity when they are connected (*a*) in series, (*b*) in parallel? (C. & G., 1931.)

4. Describe, with sketches, an electrostatic voltmeter, explaining the principle on which it acts. (C. & G., 1931.)

5. Explain the term "dielectric constant." What is the effect on the capacitance of placing a sheet of mica between two parallel insulated plates? What quantity of electricity will produce a difference of potential of 200 volts between the plates of a condenser of 5 micro-farads? Compare the capacitances of two condensers, one having two circular plates 4 cm. diameter and 1/2 mm. apart, and the other having two square plates 5 cm. each side and 3/4 mm. apart. (C. & G., 1935.)

6. Describe diagrammatically the action of the Wimshurst machine. What are the characteristics of a spark discharge and a brush discharge? (C. & G., 1930).

7. What do you understand by the potential of a body? Charges of 3, 4, 5 and -9 units are placed at A, B, C, and D respectively, the corners of a square whose sides are of length d cm. What is the potential at the middle point of AB? Find also the value of the potential if the surrounding medium is glass instead of air. (Dielectric constant of glass $= 8{\cdot}0$.)

8. A parallel plate condenser is constructed of two plates in air the distance of separation being 0·5 cm. and the area of each plate 62·8 sq. cm. Calculate the capacity of the condenser, (i) in electrostatic units and (ii) in micro-farads.

9. Explain why it is generally impossible to store a large amount of electricity on a single insulated conductor.

How is this difficulty overcome in the electrical condenser? Explain the action of the condenser. (U.E.I., S1, 1935.)

10. Find in micro-farads the capacity of a condenser which has 15 plates 4 cms. by 3 cms., the plates being separated by mica sheets of thickness 0·05 cms. (Dielectric constant of mica $= 6{\cdot}64$.)

CHAPTER IV

THE ELECTRIC CURRENT

THE ELECTRIC CURRENT.—The main theme of the previous chapter was concerned with electric charges at rest, but it was inferred that, when two conductors at different electric potentials were joined by a conducting wire, a transfer of electricity took place from the conductor at the higher to that at the lower potential.[1] This movement of electricity can be regarded as an electric current passing along the wire, but it will continue to flow only as long as a difference of potential exists between the two conductors. In the hydrostatic analogy it is evident (Fig. 58 (*a*)) that a difference of hydrostatic head is necessary to cause a flow of liquid along the connecting tube from one vessel to the other. Furthermore, just as the mean flow or current of water is measured by the volume passing divided by the time of flow, so is the *strength* (I) *of the electric current defined by the quantity* (Q) *of electricity passing per sec.*,

i.e. $I=\frac{Q}{t}$ where Q is the total quantity of electricity passing in t secs.

The practical unit of current (which will be defined later in the chapter) is called the **ampere,** after the renowned French Professor, André Ampère (1775-1836) who was a genius in both theory and experiment. The corresponding unit of quantity is the **coulomb** which *is the quantity of electricity passing when one ampere flows for one second.*

Again a constant potential difference (usually abbreviated as P.D.) may be maintained between the ends of a conductor by the use of a primary cell, in which the necessary energy is supplied by chemical action. In these circumstances the current passing across any section of the conductor will be constant provided other physical conditions remain unaltered, just as in the hydrostatic case if the difference of pressure between the ends of a tube is kept constant, then the volume of water passing through any section of the tube is always the same, even if the tube is not of uniform bore.

---

[1] It is conventional to regard the +ve direction of the current to be from the high to the low potential, but in a solid conductor it is only the −vely charged electrons which carry the charge and these move in the opposite sense. From a modern aspect therefore the true current or flow is the passage of electrons from the low to the high potential.

The question which now arises is with regard to the relationship between the P.D. and the consequent current, and the solution was found originally by a German scientist, *G. S. Ohm*. Ohm was born in 1789, the elder son of a Bavarian locksmith, and despite his reduced circumstances in early life, he managed to obtain a university education. In 1826 he formulated the generalisation known as Ohm's Law which states that **the current through any portion of a given conductor is proportional to the potential difference between its ends, provided the temperature and other conditions remain constant.** The discovery received scant recognition in his own country at the time, but its importance was soon recognised by foreign scientists, who included the Englishman, Wheatstone. At the time of his death in 1854 Ohm was a Professor at Munich University.

Ohm's Law is one of the simplest yet most important laws in electricity, and its significance will become evident from the following consideration:

If AB represents a wire and the point A on it is at a higher potential than the point B, a current whose magnitude depends upon the difference of potential will flow from A to B. If this P.D. is doubled the current will also be doubled, and, likewise, if the P.D. is trebled, the current will be exactly three times as great, it being assumed that the temperature and other conditions are constant.

Hence if I represents the current and V the P.D., then

$$\text{Current} \propto \text{P.D.}$$
$$\text{or } I \propto V$$

Therefore for a *given* conductor $\frac{V}{I}$ *is a constant.*

This constant is called the **resistance** (R) of the given conductor and represents the opposition offered to the passage of an electric current through it. Since in metallic conduction it is only the electrons that move, as even in an electric field the atoms of +ve electricity keep their positions, then it follows that the resistance will correspond to the "friction" between the electrons and the atoms of the conductor.

The practical units of I, V and R are respectively the ampere, the volt and the ohm, and they are so chosen that

$$\mathbf{I}=\frac{\mathbf{V}}{\mathbf{R}} \text{ or } \mathbf{Amperes}=\frac{\mathbf{Volts}}{\mathbf{Ohms}}$$

which is an alternative expression of Ohm's Law. It should be noted that *this law may be applied to any portion* or to the whole *of an electric circuit.*

The unit of resistance, the ohm, is usually denoted by ω (omega), and the international standard ohm is defined as the

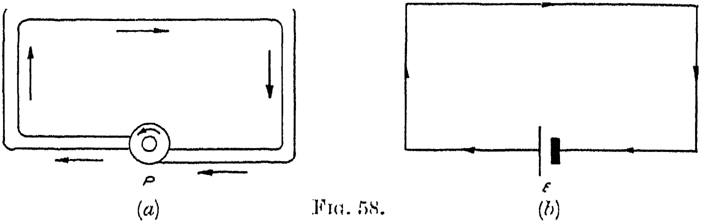


Fig. 58.

resistance of a column of mercury 106·3 cm. long, 1 sq. mm. cross-section at the temperature of melting ice. A mercury column was originally chosen as the standard since a wire resistance is liable to change with time.

Electro-motive Force.—The maintenance of a current in an electric circuit involves the motion of electrons around a continuous path and may thus be compared with the hydrostatic case given in Fig. 58 (*a*). In this case a pump P maintains a circulation of water through a complete circuit of pipe line just as a battery or generator E keeps a current flowing in the electric circuit (Fig. 58 (*b*)). It is evident that when the liquid is circulating in the water circuit that the pump itself will offer some resistance to the flow in addition to that due to the tubes in the external circuit. Similarly in the electrical case the current has to flow through the generator as well as through the external resistance. In consequence the *whole of the electrical pressure developed by the generator or battery*, and known as the *Electro-Motive Force* (**E.M.F.**), is not available for driving the current in the external circuit. Hence the *P.D. between the terminals of a cell* (i.e. terminal P.D.) *when* it is *delivering a current is less than* the *E.M.F.* (E) of the cell.

Denoting the resistance of the cell or battery by B and the external resistance by R, the current I flowing in the circuit is given by $I=\frac{E}{R+B}$ . . . . (1)

But Ohm's Law is applicable to a portion of the circuit, hence if V is the terminal P.D. of the cell when the current I is flowing, it is evident that $I=\frac{V}{R}$ . . . . . (2)

From (1) and (2) it follows that

$$IB=E-IR=E-V$$

i.e. the drop of potential in the cell=E.M.F. of the cell (E) —external drop of potential (V).

The **electro-motive force of a cell is the potential difference between its terminals when the cell is on open circuit,** i.e. when no current is flowing through it.

Example 1.—What potential difference is required to maintain a current of ½ ampere through a lamp of 420 ohms resistance ?

By Ohm's Law

$$V=IR$$
$$=\tfrac{1}{2}\times 420$$
or $V=210$ volts.

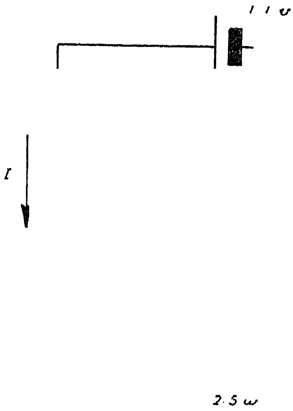


FIG. 59.

Example 2.—A cell has an e.m.f. of 1·1 volts and a resistance of 2 ohms. Its terminals are connected to a coil of resistance 2·5 ohms. What is the current through the circuit and what is the potential difference across the coil ?

E.M.F. = 1·1 volt (see Fig. 59).

Total Resistance = 2ω + 2·5ω = 4·5ω.

By Ohm's Law I (current) $=\frac{E}{R}=\frac{1\cdot1}{4\cdot5}=0\cdot24$ ampere.

Let V be the potential difference across the coil.

Then V = current through the coil × resistance of the coil

$$=\cdot24\times2\cdot5=0\cdot6 \text{ volt.}$$

MAGNETIC FIELD DUE TO A CURRENT.—The discovery that there is a magnetic field in the neighbourhood of a wire which carries an electric current, like many other important discoveries in science, was made accidentally. A relationship between magnetic and electric phenomena had long been suspected, but no one had been able to detect it. A Danish scientist, **Hans Oersted** (1777-1851), who was Professor of Physics at Copenhagen, one day after a lecture casually moved a wire carrying an electric current, over a compass needle, which happened to be freely suspended in a case on the table. The needle was deflected and Oersted found that when the current passed in the opposite direction through the wire, the deflection was also in the opposite direction. Furthermore he discovered that the interposition of substances such as stone or glass between the wire and the needle did not in any way affect the action.

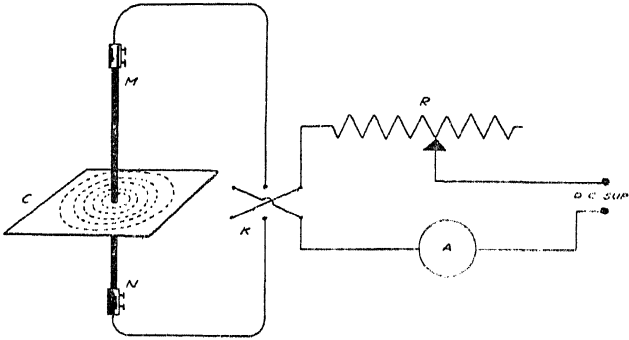


FIG. 60.

The actual form of the magnetic field in the neighbourhood of a straight conductor carrying a current can be determined as follows :

Experiment.—Pass a straight conductor MN (Fig. 60), of copper or brass, through the centre of a square piece of cardboard and connect the ends of the conductor through a reversing key, K, to the terminals of a battery as shown. When a current is passing, map out the magnetic field in the neighbourhood of MN by means of a compass needle, and verify that the lines of force are concentric circles round the axis of the conductor. Carefully note the *direction of the field* and observe that this direction is also reversed when the current in the conductor is reversed.

The above experiment which is due to Arago should also be carried out using iron filings to map the field, but it will be found necessary to use much larger currents of the order of 7 or 8 amperes.

The student should verify that the relative directions of the current flow and the resultant magnetic field as observed in the above experiment are satisfied by the following rule :

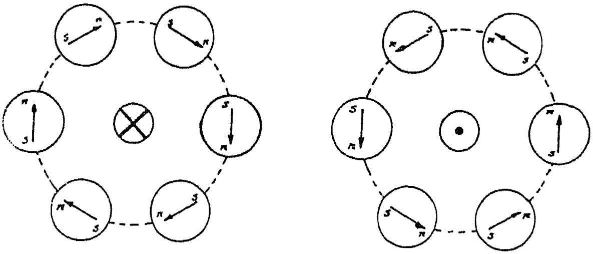


FIG. 61.

Imagine a right-handed screw being turned so that it travels in the same direction as the current. The direction in which the thumb rotates is the direction in which a free north pole is urged, i.e. the direction of the lines of force (see Fig. 61).

The above rule is known as the **Corkscrew Rule,** and is attributed to **Clerk Maxwell** (1831-1879), an English mathematical physicist who is renowned for his work on electromagnetic waves. Further rules are often quoted due to Ampère and Fleming respectively, but they are more difficult to remember than the corkscrew rule which may be applied in all cases. A useful application of the rule is the determination of the direction of a current in a wire when the source of current is inaccessible.

Experiment.—Place a small compass needle on the table and hold at a little distance above it a wire which is set in the direction of the magnetic meridian and connected through a key to the terminals of a cell. Complete the circuit and observe the direction in which the north pole is deflected. Reverse the direction of the current and again observe the direction in which the north pole is urged. Repeat the procedure when the compass needle is held above the wire. Tabulate your results as follows :

	Direction in which the north pole is urged when the wire :	
	(1) is *above* the needle.	(2) is *below* the needle.
Current flowing from north to south.		
Current flowing from south to north.		

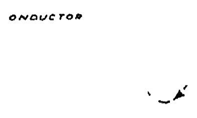


FIG. 62.

Use the above observations to test the corkscrew rule (see Fig. 62) and to deduce the effect of placing the needle at the centre of a rectangular coil carrying a current.

Further experiments to determine fields of force :

Experiment 1.—By means of a compass needle determine the distribution of the lines of force in the neighbourhood of two parallel conductors which convey currents, (*a*) in the same direction and (*b*) in opposite directions. The distribution of the lines is shown in Figs. 68 and 69.

Experiment 2.—Pass a circular coil of wire C through a sheet of wood or cardboard (Fig. 63 (*a*)), and preferably arrange the coil to be in the magnetic meridian. Connect the ends of the coil to a battery and by means of a compass needle plot the lines of force in the neighbourhood of the coil, when a current passes through it. The distribution of lines in the plane of the board is indicated in Fig. 63 (*a*), while Fig. 63 (*b*) shows the distribution in various planes. Reverse the direction of the current in the coil and it will be found that the direction of the field is reversed.

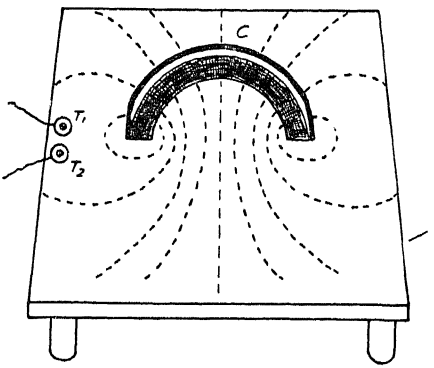


FIG. 63 (*a*).

Experiment 3.—Construct a helix (or solenoid) by winding a length of wire upon the surface of a cylinder, thus forming a number of loops side by side. Adopting a similar procedure as before, determine the distribution of the lines of force in the neighbourhood of the solenoid when a current is passing. It will be found that the lines of force arrange themselves as shown in Fig. 64.

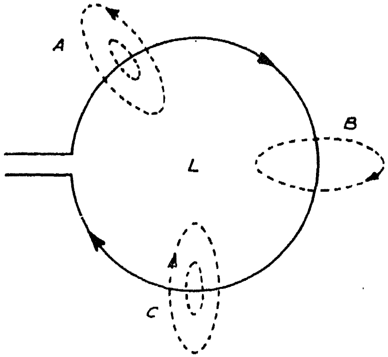


FIG. 63 (*b*).

It should be carefully noted that a helix through which a current is passing, acts in every way like a
b
h
its two ends. These polarities may be determined by an application of the corkscrew rule, but are also conveniently found by the clock rule given on page 13.

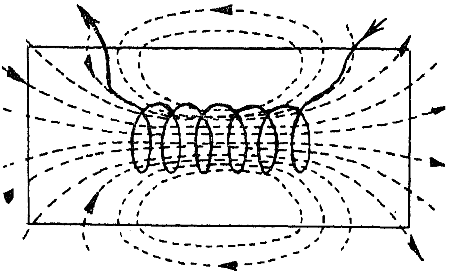


FIG. 64.

In the experiments which have just been described, evidence has been furnished of

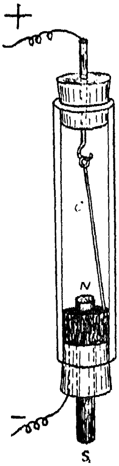


FIG. 65.

the effect of a current on a magnetic needle, and also on the iron filings which become small temporary magnets when they are placed in the magnetic field created by the current. Now according to Newton's Third Law of Motion (page 16) to every action there is an equal and opposite reaction, hence it is logical to expect that a magnet will exert a force on a conductor carrying a current in its neighbourhood. This fact can easily be demonstrated.

Experiment 1.—Suspend a thick brass wire C from a hook as shown in Fig. 65, so that one end of it dips in mercury contained in a glass tube, while the other end is connected through the hook to a reversing key and thence via the battery and resistance to the mercury. A bar-magnet is inserted through the cork which closes the lower end of the glass tube, so that one of its poles protrudes slightly above the surface of the mercury. When a current is passed through the wire, the latter rotates round the magnet pole. If the direction of the current is reversed, the direction of rotation is also reversed. A cylindrical cobalt-steel magnet serves admirably for this experiment which was performed initially by Faraday.

Experiment 2.—Pass a current through a wire which is suspended so that it can move freely between the poles of an excited electro-magnet. (If an electro-magnet is not available two strong bar-magnets laid horizontally, with opposite poles facing each other at a small distance apart, may be employed.) It will be found that the wire tends to be displaced in a direction which is at right angles, both to the direction of the magnetic field and to the direction in which the current flows.

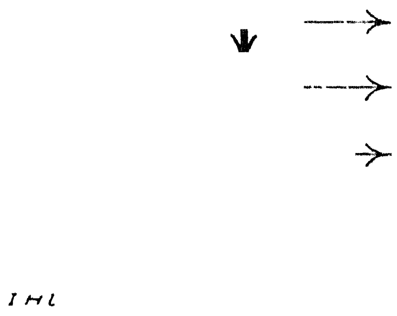


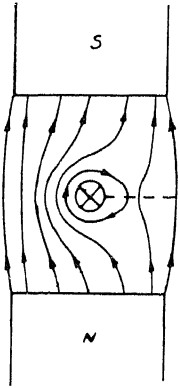
FIG. 66.

The student should

verify that the direction in which the wire is urged in the above experiment is that given by applying **Fleming's Left Hand Rule,** which is as follows :

*Place the thumb and first two fingers of the left hand mutually at right angles to each other. If the **F**irst finger points in the direction of the lines of **F**orce, and the se**C**ond finger gives the direction of the **C**urrent, then the thu**M**b will indicate the direction of **M**otion, i.e. of the resultant force on the conductor.*

If the wire is situated in a uniform magnetic field of strength H c.g.s. units and whose direction is at right angles to the wire (Fig. 66), then **the magnitude of the force is given by lIH dynes.** l is the length of the conductor considered and I (c.g.s.) is the value of the current passing (1 amp. $=0 \cdot 1$ c.g.s.).



(a) Fig. 67. (b)

The effects observed in the last two experiments may be explained as the result of the interaction between the main magnetic field and that due to the current in the conductor, in the following manner. Fig. 67 (*a*) shows the field due to a permanent magnet system (in full lines) and that due to a current-carrying conductor (in dotted lines). Under the conditions of Experiment 2 these two fields are superimposed, and the form of the resultant field is indicated in Fig. 67 (*b*). The combined effect results in a crowding of the lines of force on the left hand side of the conductor, and a weakening of the field strength (since the strength of a magnetic field is measured by the number of lines of force crossing unit area) on the right hand side. Consequently the tendency of the lines of force to shorten themselves will cause the conductor to be driven towards the right.

MUTUAL FORCE BETWEEN TWO CURRENT-CARRYING CONDUCTORS.—It is evident that a current-carrying conductor in the presence of a second conductor, which is also conveying a current, is comparable to the case of the conductor in a magnetic field which was considered in the preceding paragraph. The combined fields due to parallel straight conductors are shown in Figs. 68 and 69, and it is easily deduced from an inspection of these distributions that attraction ensues between parallel conductors (if free to move) carrying currents in the same direction, while repulsion occurs if the currents are in opposite directions. These facts may also be inferred by considering the action of the magnetic field due to the current in A on the conductor B and hence applying the left-hand rule to determine the direction of the

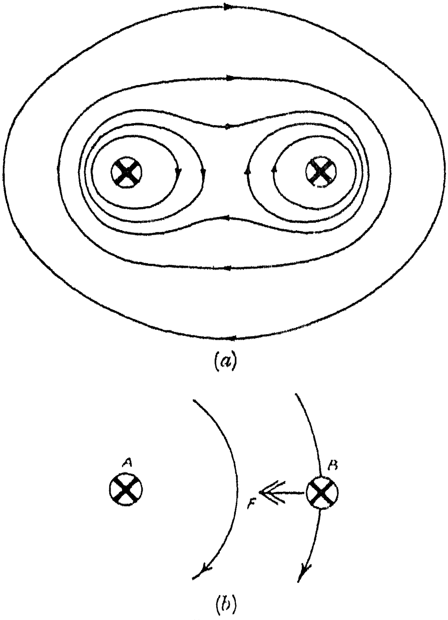
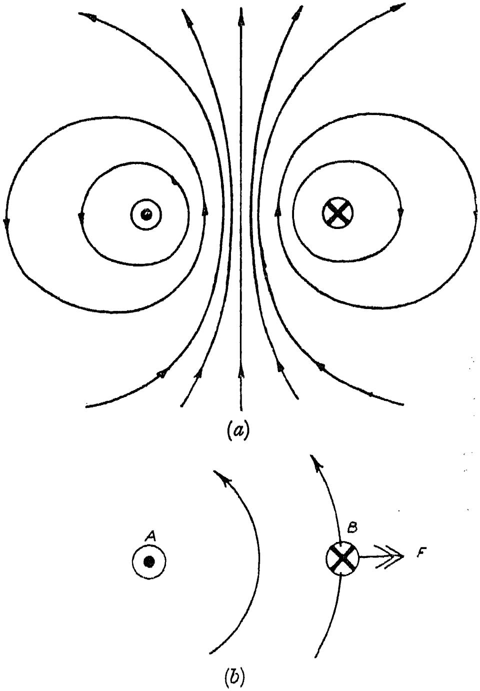


FIG. 68.

resultant force (see Figs. 68 (*b*) and 69 (*b*)). The magnitude of the effect is very small, but may be considerably enhanced if the conductors take the form of coils containing a large number of turns.

Experiment 3. *Barlow's Wheel.*—This consists of a star-shaped copper disc W (Fig. 70), which is mounted on an axle so that it can rotate between the poles of a horseshoe magnet. The metal support of the axle is connected to a terminal on the baseboard, and one of the radial points of the disc dips into a pool of mercury which is connected to a second terminal. When the terminals are joined to a battery a current passes through the "strip" which dips in the mercury, and as a portion of it is lying at right angles to the field of a horseshoe magnet the "strip" experiences a force which carries it out



(*b*)
FIG. 69.

of the mercury. An adjacent "strip" is thus brought into contact with the mercury and in turn is immediately expelled from it, by the action of the magnetic field. As a consequence of these actions the wheel receives a continuous succession of impulses so that it acquires a rotational motion. The direction of this motion can be found by applying Fleming's Left

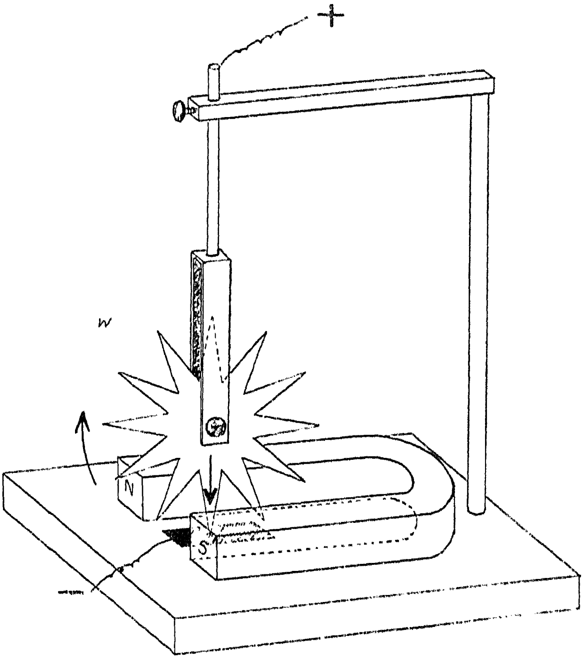
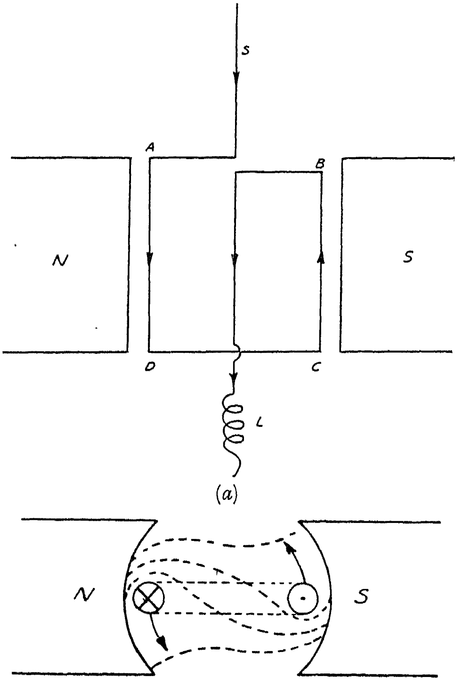


FIG. 70.

Hand Rule (page 73). If the current is reversed the direction of the rotation is also reversed.

Experiment 4.—Suspend a light coil of wire ABCD (Fig. 71 (*a*)) by a fine wire S between two closely fitting poles of a powerful curved magnet (or preferably an electromagnet). A current can be circulated in the coil by connecting a battery to the upper end of the suspension and to the loose spiral L. When a current passes in the coil the latter tends to set itself at right angles to the direction of the magnetic field, the direction of motion being in accordance with Fleming's Left Hand Rule (page 73). This tendency of the coil to embrace the greatest possible number of lines of force is resisted by the twist imparted to the suspension as the coil turns. The *coil will move until the controlling torque* (or couple) *due to the torsion of the suspension is equal to the deflecting couple exerted on the coil by the magnetic field.*



(*a*)

(*b*)

Fig. 71.

The deflection can be observed conveniently by means of a light pointer attached to the suspension, and this deflection will be found to increase with the current flowing through the loop. It should be noted, however, that if the coil was suspended *quite* freely it would move through 90° independently of the value of the current in the loop.

As the last experiment is an important illustration of the action of certain measuring instruments it is necessary to consider briefly how the deflecting couple is exerted on the coil. For this purpose refer to Fig. 71 (*b*), which is a plan of the system represented in Fig. 71 (*a*). (It should be noted here that the conventional signs for currents coming out and going into the paper are respectively ⊙ and ⊗.) Since the horizontal limbs of the suspended coil *lie along* the direction of the magnetic field (strength H c.g.s. units), the only forces to be considered are those on the vertical members AD and BC (each of length l cm.). The direction of these two forces will be given by the left hand rule, and the magnitude F, of either, by $\mathrm{IH}l$ dynes (page 73), where I is the current (in c.g.s.) flowing in the coil. It is evident that these forces will produce a couple tending to rotate the coil about the axis of the suspension, and its magnitude will be given by

$\mathrm{F} \times x = \mathrm{IH}l \times x$ (x being equal to the length AB),
i.e. IHA, where A is the area enclosed by the coil.

An alternative method of regarding the deflection of the coil is to remember that on the passage of a current through the loop, its opposite faces develop respectively north and south polarity, as governed by the clock rule (page 13). Consequently, so far as magnetic action is concerned the current carrying coil can be considered to be replaced by a short bar-magnet, with its axis perpendicular to the plane of the coil. This bar-magnet will obviously tend to set along the lines of force, i.e. the line of the poles, which therefore means that the coil will endeavour to place itself perpendicular to this direction.

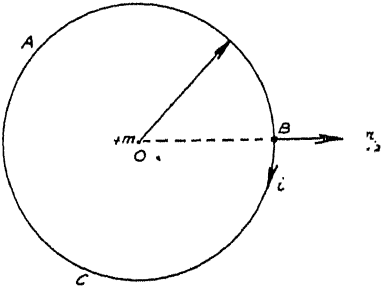


FIG. 72.

THE STRENGTH OF THE FIELD AT THE CENTRE OF A CIRCULAR COIL CARRYING A CURRENT.—Let ABC (Fig. 72) represent a circular coil of wire, of radius r, and carrying a current I c.g.s. units. To measure the strength of the field suppose a magnet pole of strength m to be situated at the centre of the coil. Now the magnetic field due to this pole in the plane of the loop will be *everywhere* normal to the conductor ABC, e.g. at B it will act along the radius OB. The magnitude of the force due to the pole at a distance r from the centre is given by $\frac{m}{r^2}$ (page 20).

Hence by the rule on page 73 the force on the conductor $\mathrm{ABC} = \mathrm{IH}l = \mathrm{I}\frac{m}{r^2}\,2\pi r$ (since l = circumference of coil), i.e. force $=\frac{2\pi Im}{r}$ and the direction of the force will be perpendicular to the plane of the loop as given by the left hand rule. But by Newton's Third Law of Motion, action and reaction are equal and opposite, hence the force exerted on the pole at O will be $=\frac{2\pi Im}{r}$.

It follows that the force on unit pole at O, which is the measure of the field strength due to the current I in the coil, is given by $\frac{2\pi Im}{r}\div m$, i.e. $\frac{2\pi I}{r}$ dynes, and furthermore this force will act normal to the plane of the coil at O, i.e. along its horizontal axis.

It is obvious that if the coil has n turns the field will be n times as great, i.e. $=\frac{2\pi nI}{r}$.

It follows from the above expression for the intensity of the magnetic field at the centre of a circular coil carrying a current I c.g.s., i.e. $F=\frac{2\pi I}{r}$, that if $r=1$ cm. and $I=1$ c.g.s. unit of current, then $F=2\pi$ dynes.

Hence the unit current in the electro-magnetic system of units is defined as that current which, flowing in a circular conductor of unit radius, produces at the centre a magnetic field of 2π gauss.

In practice the E.M.U. of current is found to be inconveniently large and a smaller unit, called the **ampere,** is taken as the practical unit of current, such that

1 E.M.U. of current = 10 amperes.

QUESTIONS.

1. State Ohm's Law. Determine what potential difference is required to light a lamp of resistance 110 ohms which takes a current of 0·5 ampere.

2. State the Corkscrew Rule for determining the direction of the electric current flowing in a wire. Give an account of an experiment which illustrates it.

3. Describe an experiment which shows that a magnet exerts a force on a current-carrying conductor in its neighbourhood.

4. A long wire, through which a current can be sent is placed in a North and South direction. A small magnetic needle is placed a short distance *below* the wire. In what direction will the needle be deflected when a current is passed through the wire from south to north? What will be

the effect on the magnitude and direction of the deflection of: (*a*) reversing the current, (*b*) increasing the current, (*c*) substituting another magnetic needle of exactly the same size, but with poles of greater strength, and (*d*) placing the magnetic needle above instead of below the wire?

You should assume that the needle has had time to come to rest in each case. (C. & G., 1929.)

5. Describe and explain the action of Barlow's Wheel.

6. Describe and explain what happens when an electric current is passed through a loop of wire which is suspended by a torsion fibre so that it hangs freely between the poles of a powerful magnet.

CHAPTER V

MAGNETIC MEASUREMENTS

MAGNETIC MOMENT.—It has been seen that if a magnet is pivoted so that it can oscillate freely in a horizontal plane it will come to rest with its axis in the magnetic meridian. Consider the case of a magnet that is deflected out of the meridian (Fig. 73). Let m be the pole strength of the magnet, $2l$ the distance between the poles, and H the *uniform* magnetic intensity due to the earth's field acting in the horizontal plane in which the magnet rests. It is evident that the magnet is acted upon by a restoring couple which urges it back into the meridian.

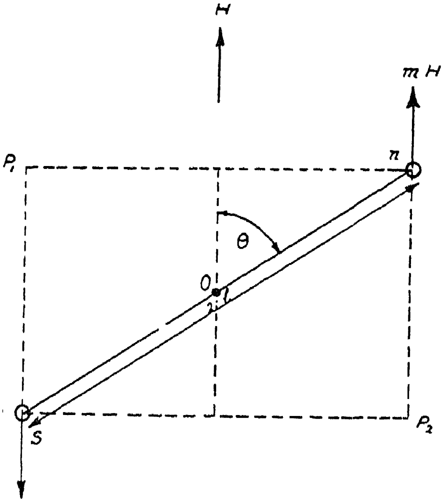


FIG. 73.

The *moment of this restoring couple*

$$\begin{aligned} &= m \,.\, \mathrm{H} \times np_1 \\ &= m \,.\, \mathrm{H} \times 2l \sin \theta \\ &= 2\, ml \,.\, \times H \sin \theta \end{aligned}$$

If $\theta=90°$ and H is unity, then since $\sin 90°=1$, the restoring couple is equal to $2\,ml$. Hence in order to maintain a magnet perpendicular to a field of unit intensity a couple must be applied equal in magnitude to the product of the pole strength (m) and the distance ($2l$) between the poles. This product is called the **Magnetic Moment** of the magnet, and is usually denoted by M.

$$\text{i.e. } \mathbf{M}=m\times 2l=\mathbf{2lm}.$$

(The length between the poles is taken as $2l$ for convenience in further mathematical analysis of magnetic problems. It should also be noted that north and south poles are considered as +ve and —ve respectively.)

The Magnetic Moment of a magnet is numerically equal to the moment of the couple acting on it when it is held perpendicular to a uniform field of unit intensity.

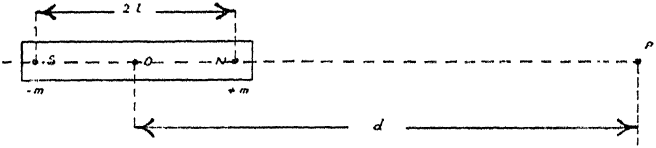


FIG. 74.

DETERMINATION OF THE INTENSITY OF THE MAGNETIC FIELD DUE TO A BAR-MAGNET (SITUATED IN AIR).

(1) *At a point on the axis produced of the magnet.*—Consider any point P on the axis produced of the magnet SN (Fig. 74) and at a distance d from the centre of the magnet. Let $m=$ pole strength and $2l=$distance between the poles of the magnet.

To measure the magnitude of the magnetic field at P due to the bar-magnet, a unit north pole is supposed to be placed at that point.

Then the force on this unit pole due to the north pole of the magnet is by a previous result (page 20) $=\dfrac{m}{(d-l)^2}$ dynes in the direction OP (a repulsive force).

Similarly the force on unit pole at P due to south pole of magnet$=\dfrac{m}{(d+l)^2}$ dynes in the direction PO (an attractive force).

∴ *Resultant Force* (a repulsion) in the direction OP

$$=\frac{m}{(d-l)}\quad\frac{m}{(d+l)^2}=\frac{4\,mld}{(d^2-l^2)^2}\ \textit{dynes}$$

But the Magnetic Moment (M) of the magnet $=2\,ml$.

∴ *Intensity* of magnetic field at P $=\frac{2Md}{(d^2-l^2)^2}$ *gauss.*

This expression may be written as $\frac{2M}{d^3\left(1-\frac{l^2}{d^2}\right)^2}$ and it is easily seen that if l is *small* compared with d, then l^2 is negligible in comparison with d^2 and the expression reduces to the approximate form $\frac{2M}{d^3}$ gauss.

The error involved by using the approximate formula should be calculated by the student for various values of l/d.

Note.—In this position the magnet is said to be END-ON to the point P.

(2) *At a point in the equatorial plane of a magnet.*—Referring to Fig. 75 and using the notation of the previous case.

The force on unit north pole placed at P

(*a*) due to north pole of magnet

$$=\frac{m}{(NP)^2}=\frac{m}{(d^2+l^2)}\ \text{dynes}$$

in the direction PN¹ (a repulsion)

and (*b*) due to south pole of magnet

$$=\frac{m}{(SP)^2}=\frac{m}{(d^2+l^2)}\ \text{dynes in}$$

the direction PS¹ (an attraction).

If PN¹ and PS¹ are drawn to represent these forces on a particular scale, then their resultant force (*R*) will be given by the diagonal PR of the parallelogram. Since PN¹ = PS¹, then PR is

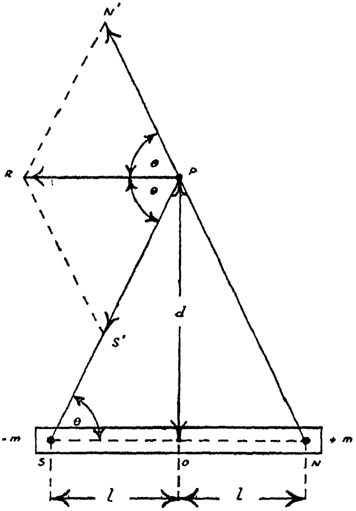


FIG. 75.

parallel to SN, and it follows that

$$R=2\times\frac{m}{d^2+l^2}\cdot\cos\theta=\frac{2m}{d^2+l^2}\cdot\frac{SO}{PS}=\frac{2ml}{(d^2+l^2)^{3/2}}\text{ dynes}$$

or *Intensity of Magnetic Field* at P is $=\frac{M}{(d^2+l^2)^{3/2}}$ gauss in a direction parallel to the axis of the magnet.

The expression may be written alternatively as

$\frac{M}{d^3\left(1+\frac{l^2}{d^2}\right)^{3/2}}$ gauss, whence it is evident that if l^2 is *small* compared with d^2 the expression reduces to $\frac{M}{d^3}$ gauss.

Note.—In this position the magnet is said to be BROADSIDE-ON to the point P.

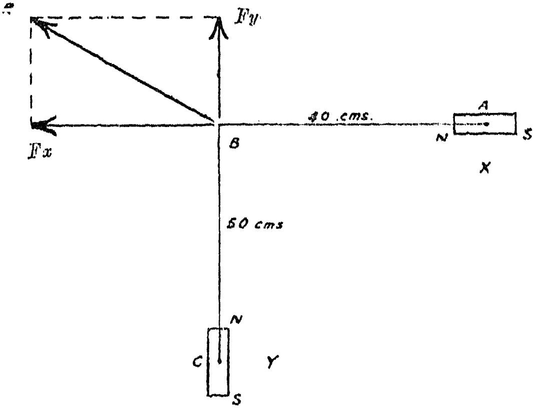


FIG. 76.

Example.—Two straight lines AB and BC of lengths 40 and 50 cms. respectively are drawn on a table perpendicular to each other. Two *short* bar-magnets, X and Y, of magnetic moments 50 and 150, are placed along these lines with their centres at A and C respectively. Determine the intensity of the combined field at B (Fig. 76).

Force on unit north pole at B due to X is

$$F_x=\frac{2M}{d^3}=\frac{2\times50}{40^3}=\frac{1}{640}\text{ dyne.}$$

Similarly force on unit north pole at B due to Y is

$$F_Y = \frac{2 \times 150}{50^3} = \frac{3}{1250} \text{ dyne.}$$

By the parallelogram of forces the resultant force R is given by $R = \sqrt{F_X^2 + F_Y^2} = 0{\cdot}00286$ *dyne.*

Hence the intensity of the combined field at B is 0·00286 *gauss.*

The student should carefully note that the *magnitude* of the resultant intensity will be unchanged if *either* magnet is reversed pole for pole.

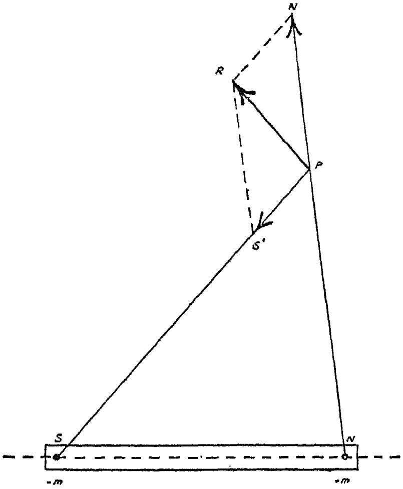


FIG. 77.

GRAPHICAL METHOD OF DETERMINING THE FIELD STRENGTH AT ANY POINT IN THE NEIGHBOURHOOD OF A BAR-MAGNET.—Referring to Fig. 77 it is required to find the intensity of the magnetic field at any point P in the neighbourhood of the bar-magnet SN.

The following data is given :

SN=4 cms., NP=7 cms. and SP=9 cms. $m=20$ unit poles.

Then force on unit north pole at P due to the north pole of magnet is $\frac{20}{7^2}$ dynes in direction PN^1 and the force on unit north pole at P due to south pole of magnet is $\frac{20}{9^2}$ dynes in direction PS^1.

To obtain the resultant of these forces at P mark off a distance PN^1=a chosen unit of length, i.e. if $\frac{20}{49}$ dynes is=1 unit of length.

$$\text{Then } \frac{20}{81} \text{ dynes will be} = \frac{\frac{20}{81}}{\frac{20}{49}} \text{ i.e. } \frac{49}{81} \text{ units of length.}$$

Hence mark off a distance $PS^1 = \frac{49}{81}$ units of length.

Complete the parallelogram and measure PR, which gives the direction and magnitude (according to the scale chosen) of the resultant magnetic force at P.

By measurement, PR=·531 unit of length. But 1 unit of length is equivalent to a force of $\frac{20}{49}$ dynes.

Hence *Resultant Force at P* is

$\frac{20}{49} \times \cdot531$ dynes per unit pole $= 0{\cdot}215$ *dynes per unit pole.*

i.e. *Intensity of Magnetic Field at P* due to bar-magnet is 0·215 *gauss.*

By measurement with a protractor the direction of the field at P makes an angle of 103° with the direction of the magnetic axis $\overrightarrow{SN}$ of the bar-magnet.

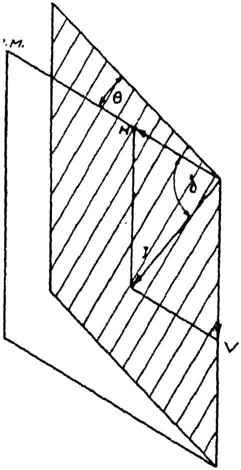


Fig. 78.

Terrestrial Magnetism.—Earlier in this book it was indicated that the magnetic meridian did not exactly coincide with the geographical meridian. The **angle** θ (Fig. 78) **between** the planes of the **geographical** (shaded in figure) **and magnetic meridians** at any place on the earth's surface is defined as the **Declination** at that place. The angle of declination varies considerably over the earth's surface. In London, for example, it is 11° 40′ *west* of geographical north, while in California it is about 20° *east* of geographical north. At some places the declination is zero, which means that the two meridians are coincident.

Suppose now that a *balanced* needle (i.e. a needle pivoted at its centre of gravity) is carefully magnetised and pivoted so that it can oscillate freely in a *vertical* plane, in fact an instrument which is known as a Dip Needle or Dip Circle. Furthermore, if this vertical plane is made coincident with the magnetic meridian it will be noted that the needle comes to rest with its axis at an angle to the horizontal. This angle ((delta) in Fig. 78) is called the angle of dip or the inclination at the place of observation.

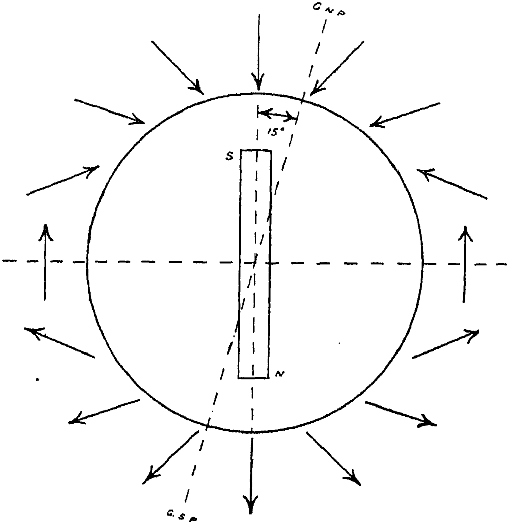


FIG. 79.

The angle that the axis of a magnetic needle (free to oscillate in a *vertical* plane) **makes with the horizontal at any place, when the plane of the needle is in the magnetic meridian, is called the inclination or angle of dip at that place.**

The value of the dip in London at the present time is 66° 41′, the north pole of the needle pointing downwards, but at places near the equator it is zero while in the southern hemisphere the south pole dips downwards.

Sir William Gilbert, one of Queen Elizabeth's physicians, in 1600 suggested that the explanation of the above facts was consistent with the postulate that the earth itself formed a large magnet. The general characteristics of the earth's magnetic field may be realised approximately by assuming a large magnet to be situated near the centre of the earth, and inclined at an angle of approximately 15° to its geographical axis (Fig. 79). The above model does not enable the irregularities of the earth's magnetism to be explained, but it is very useful as indicating the variation of magnetic dip over the earth's surface. The arrowheads in Fig. 79 indicate the directions that would be assumed by the north pole of a dip needle when placed at various points on the earth's surface. It is evident that the angle of dip is zero at the *magnetic equator*, and increases to 90° as the north and south magnetic poles are approached. The position of the north magnetic pole was discovered by Sir James Ross in 1831 near Borthia Felix in British North America, while the south magnetic pole was located during Sir Ernest Shackleton's expedition in 1909.

The direction assumed by a dip needle at any place *when the plane of the needle is in the magnetic meridian*, gives the direction of the resultant or total intensity (I) of the earth's magnetic field at that place. In Fig. 80 (also Fig. 78), δ denotes the angle of dip and H and V the horizontal and vertical components of the earth's field respectively. It is evident that the following relations hold: $\cos \delta = H/I$, $\tan \delta = V/H$ and $I = \sqrt{V^2 + H^2}$.

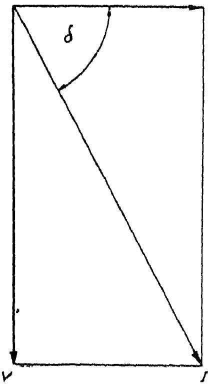


Fig. 80,

Example.—If the value of H at a certain place is 0·195 gauss and the angle of dip is 62°, find the value of the vertical component (*a*) graphically, and (*b*) by calculation. Use your result to determine the total intensity (I) at the given place ($\tan 62° = 1{\cdot}881$).

(*a*) *Graphically.*—Draw AH (Fig. 80) a certain length to represent the earth's horizontal component (H) and suppose the scale is chosen so that 1 in. represents 0·100 gauss, then AH=1·95 in. Next draw a line AI making an angle $\delta = 62°$

with AH and meeting a line perpendicular to AH at I. Complete the rectangle by drawing IV parallel to HA and AV parallel to HI.

Measure AV which represents the value of the vertical component (V) on the scale chosen. By measurement AV= 3·68 in. and $\therefore$ V=0·368 gauss.

(*b*) *By calculation.*—If δ is the angle of dip then $\tan \delta = V/H$, i.e. $V = H \tan \delta = 0{\cdot}195 \times \tan 62^\circ = 0{\cdot}195 \times 1{\cdot}881 = 0{\cdot}367$ gauss.

To determine the value of I the relation $I = \sqrt{V^2 + H^2}$ is employed, hence $I = \sqrt{0{\cdot}367^2 + 0{\cdot}195^2} = 0{\cdot}415$ gauss.

The horizontal component H is clearly of great importance in practice, for this is the force to which magnets are subjected when they are suspended or pivoted so that they lie in a horizontal plane. As in the case of the angle of dip the value of H varies from place to place and its value at any place varies with time. In London at the present time H=0·185 c.g.s., V=0·430 c.g.s., and I=0·468 c.g.s.

H, V, I, δ and D are known as the **magnetic elements** of the earth's field and three of these magnitudes, one of which must be D, are necessary to specify completely the earth's magnetic field at any place. In nautical and geographical work the values of H, δ and D are usually recorded. Magnetic maps are drawn in which places on the earth's surface having the same value for the magnetic elements are joined together by lines. *Isogonals* are lines joining places of equal declination, *isoclinals* connect places of equal dip and *isodynamic* lines join places of equal horizontal force.

An exact knowledge of the declination at any place and time is of the utmost importance to navigators, and charts can be prepared for their use as fortunately the change of declination with time is quite regular, and may be estimated accurately beforehand. By way of interest it may be mentioned that the daily changes of declination are found to be a maximum when the sun-spots are most prominent.

The *Mariner's Compass* enables the navigator to determine the magnetic meridian at the place of observation, so that by the use of the declination chart he is able to locate his position. The form of compass which is most widely employed to-day is that due to Lord Kelvin. It consists essentially of a number of light magnetic needles which are fixed to the underside of a circular card. This card, which has the thirty-two points of the compass marked upon it, is made of some light non-magnetic material, such as aluminium or mica. It is pivoted at its centre so that it can rotate easily, and in modern instruments the "card" is immersed in a liquid, usually methylated spirits, contained in a copper bowl. This arrangement has the dual advantage of diminishing the pressure on the support, and keeping the "card" from oscillating excessively. By using a "ladder" formation of some eight light needles Kelvin enhanced the magnetic moment of the system without unduly increasing the weight, and thus increased the sensitivity of the instrument.

EFFECT OF THE SHIP'S MAGNETISM.—The magnetising action of the earth's field, to which reference has been made, obviously introduces complications in the use of the compass in a steel ship. The latter acquires permanent magnetism during the process of construction, and, in addition to this, it also becomes temporarily magnetised in a manner which varies with the position and direction of the ship. The errors which would be introduced in the compass readings, both by the temporary and permanent magnetisation, are eliminated by placing soft iron spheres and permanent magnets in appropriate places in the neighbourhood of the compass.

MAGNETIC MEASUREMENTS

(*a*) *Magnet oscillating in a uniform magnetic field.*—Consider a suspended magnet which has been deflected through an angle θ from its position of rest NS in a uniform horizontal magnetic field of intensity H gauss. On the removal of the deflecting force the magnet will be acted upon by the restoring couple due to the magnetic field (page 81), and will return towards its position of rest, but in consequence of its kinetic energy will overshoot this mark. The magnet will continue its motion until it reaches a position which makes an angle with NS *nearly* equal to θ, when it will again commence to return towards its "zero." This *oscillatory* movement of the magnet will continue, but the *amplitude* of vibration (defined by the angle of deflection θ) will become smaller and smaller. It is important to note, however, that the *period* of the vibration, i.e. the time required to perform a back *and* fro movement, remains constant and is given by $t=2\pi\sqrt{\frac{I}{MH}}$ secs. where I=moment of inertia (in c.g.s.) of the magnet about the axis of the suspension and M=magnetic moment of the magnet.

In performing experiments with oscillating magnets, it is important that the magnets should be protected from draughts during the time that observations are made. For this purpose a wooden box provided with sliding glass doors in the back and front is usually employed.

TO COMPARE THE STRENGTHS OF MAGNETIC FIELDS.—If a magnet is allowed to oscillate freely in two fields of strength, H_1 and H_2, then using the previous notation

$$t_1=2\tau \qquad MH_1 \quad \text{and} \quad t_2=2\pi\sqrt{\frac{I}{MH_2}}. \qquad \text{Hence } \frac{t_1}{t_2} = \sqrt{\frac{H_2}{H_1}}$$

where t_1 and t_2 are the respective periods.

Now if n is the number of complete vibrations performed in 1 sec., then $t=1/n$, i.e. the period is *inversely* proportional to the frequency (n), and it follows that $t_1/t_2=n_2/n_1$ where n_1 and n_2 are the frequencies corresponding to t_1 and t_2 respectively.

$$\text{Consequently } \frac{n_2}{n_1} = \frac{t_1}{t_2} = \sqrt{\frac{H_2}{H_1}} \text{ or } \frac{H_1}{H} = \frac{n_1^2}{}$$

Since the period of vibration of the usual laboratory magnet is quite long it is more convenient to let n_1 and n_2 represent the number of vibrations per minute.

A small length of magnetised knitting needle suspended in a beaker is a simple form of *vibration magnetometer,* suitable for finding the variation of field strength at different parts of a room or building.

TO COMPARE THE MAGNETIC MOMENTS OF TWO MAGNETS. *Experiment.*—(1) The *moments of inertia* of the magnets are *equal.*

Oscillate each magnet in turn in the *same* magnetic field and find their respective periods t_1 and t_2.

$$\text{Then } t_1/t_2 = 2\pi\sqrt{\frac{I}{M_1H}}\Big/ 2\pi\sqrt{\frac{I}{M_2H}} = \sqrt{M_2/M_1}$$

$$\text{or } M_2/M_1 = t_1^2/t_2^2$$

(2) The *moments of inertia* of the magnets are *unequal.* Take a cylindrical and a rectangular bar-magnet and fix them in a cork (Fig. 81) which is suspended in an oscillation box.

Determine the period of vibration of the system when (*a*) the like poles of the magnets are together (period t_1, say) and (*b*) the unlike poles are together (period t_2, say).

$$\text{Then } t_1=2\pi\sqrt{\frac{I}{(M_1+M_2)H}} \text{ and } t_2=2\pi\sqrt{\frac{I}{(M_1-M_2)H}}$$

$$\text{It follows that } t_1^2/t_2^2=(M_1-M_2)/(M_1+M_2)$$

$$\text{or } M_2/M_1=(t_2^2-t_1^2)/(t_2^2+t_1^2)$$

It should be noted that in these two cases the *sum* of the separate moments of inertia is constant.

Example.—Compare the field strengths at two places A and B if the same magnet performs 50 oscillations in 25 seconds at A and the same number of oscillations in 35 seconds at B.

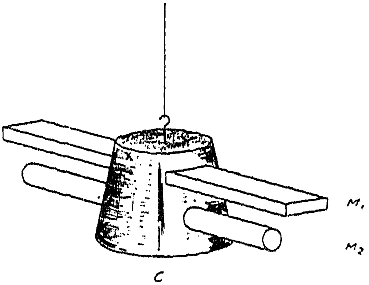


FIG. 81.

Let n_1 and n_2 be the frequencies, and H_1 and H_2 the corresponding field strengths at A and B respectively.

$$\text{Then } n_1=50/25=2 \text{ and } n_2=50/35=10/7.$$

$$\therefore H_1/H_2=n_1^2/n_2^2=2^2/(10/7)^2=1{\cdot}96.$$

Example.—A rectangular bar-magnet is 10 cms. long and 2 cms. broad and weighs 40 grams. When the magnet is suspended so that it swings in a horizontal plane in a field whose strength is 0·2 gauss, it performs 10 oscillations in 3·5 minutes. Calculate the magnetic moment of the magnet.

$$\text{Now } t=2\pi\sqrt{\frac{I}{MH}} \text{ or } M=\frac{4\pi^2 I}{t^2 H}$$

For a bar-magnet $\mathrm{I}=m\left(\frac{l^2+b^2}{12}\right)$ and

cms., $b=2$ cms. and $m=40$ grams.

$$\therefore \mathrm{I}=40\left(\frac{10^2+2^2}{12}\right)=\frac{1040}{3}\text{ c.g.s. and } \mathrm{t}=\frac{3\cdot5\times60}{10}=21\text{ secs.}$$

$$\text{Hence } \mathbf{M}=\frac{4\pi^2\times1040}{21^2\times0\cdot2\times3}=\mathbf{155\cdot3\ c.g.s.}$$

THE TANGENT LAW.

Let *ns* (Fig. 82) be a short magnetic needle which is freely suspended and with its axis in the magnetic meridian. If a bar-magnet whose axis is east and west is placed on its western side with the north pole of the magnet pointing towards the needle, the latter will be deflected into some such position as *ns*. Suppose that it is deflected through an angle θ. Let the intensity of the deflecting force be F. Then, if m is the pole-strength of the needle, the force acting on each pole is mF. The moment of the deflecting force acting on the needle is thus

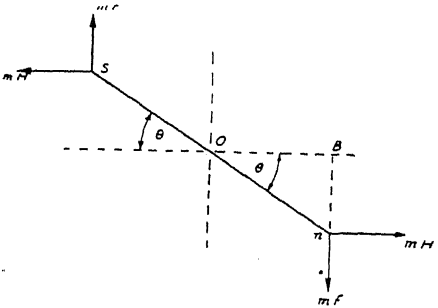


FIG. 82.

$$=2\,m\mathrm{F}\times\mathrm{OB}$$
$$=m\mathrm{F}\times2l\cos\theta$$

where $2l$ is the length of the magnet.

If H is the horizontal component of the earth's magnetic field, the moment of the restoring force acting on the needle due to the earth's field is $2m\,\mathrm{H}\times n\mathrm{B}$, i.e. $m\mathrm{H}\times2l\sin\theta$.

When the needle is in equilibrium the moment of the deflecting force is equal to the moment of the restoring force,

$$\text{i.e. } m\mathrm{F}\times2l\cos\theta=m\mathrm{H}\times2l\sin\theta$$
$$\text{or } \mathbf{F}=\mathbf{H}\ \mathbf{tan}\ \theta.$$

Now H is constant, hence $\tan\theta$ is $\propto$ F,

i.e. *the tangent of the angle of deflection is proportional to the deflecting force.*

THE TANGENT MAGNETOMETER consists of a short magnetic needle pivoted or suspended at the centre of a circular scale, which is situated in a box provided with a glass cover. A long light aluminium pointer is fixed at right angles to the needle at its centre, and the ends of the pointer travel over a circular scale when the magnet is deflected. The distance of a deflecting magnet or pole from the centre of the pivoted magnet is read on a graduated scale.

To verify the Inverse Square Law.

Experiment.—The magnetometer is first set so that its arms point east and west. A Robison ball-ended magnet is then held vertically in a wooden stand and allowed to rest on one of the arms with its pole at a known distance from the centre of the scale. The effect of the upper pole on the needle, although quite small with a long magnet, may be completely eliminated by arranging so that it is situated vertically above the centre of the needle, an adjustment which is facilitated by the use of a plumb line. Let the strength of the deflecting pole be m, and its distance from the centre of the needle be d. Then provided the needle is sufficiently small, the intensity of the field at the poles of the needle is m/d^2, according to the Inverse Square Law. Hence by the tangent law this deflecting force is proportional to the tangent of the angle of deflection, i.e. $\frac{m}{d^2} \propto \tan\theta$, or $m \propto d^2 \tan\theta$. But m is a constant and hence $d^2 \tan\theta$ should be a constant if the Inverse Square Law is true. Tabulate your results as below, taking care to read *both* ends of the pointer.

d cms.	d^2	Readings for $\theta°$ I	II	$\tan\theta$	$d^2 \tan\theta$

The student should also verify that a plot of $\frac{1}{d^2}$ against $\tan\theta$ yields a straight line.

TO COMPARE THE MAGNETIC MOMENTS OF TWO MAGNETS, BY MEANS OF THE MAGNETOMETER.

Experiment.—Set up the magnetometer with its rule running east and west, and place one of the magnets with its axis parallel to this direction and its centre distant d from the needle. Then using the previous notation (page 83) the deflecting force is $2M_1d/(d^2-l_1$ and if θ_1 is the deflection produced,

$$\frac{2M_1d}{(d^2-l_1^2)} = H \tan \theta_1 \quad . \quad . \quad . \qquad (1)$$

Repeat the observations for the second magnet placed at the same distance from the needle.

Then
$$\frac{2M_2d}{(d^2-l_2^2)} = H \tan \theta_2 \quad . \quad . \qquad (2)$$

From (1) and (2) it follows that

$$\frac{M_1}{M_2} = \frac{(d^2-l_1^2)}{(d^2-l_2^2)} \; \frac{\tan \theta_1}{\tan \theta_2}$$

Use this method to compare the moments of the same magnets as used in the vibration experiment.

MAGNETIC INDUCTION.—In a magnetised specimen of iron, steel or other magnetic substance the **intensity of magnetisation** (I) *is defined as* **the pole strength per unit area of cross section.**

Thus for any uniformly magnetised body it follows that

$$I = \frac{\text{Magnetic Pole Strength}}{\text{Area of Cross Section}} = \frac{m}{}$$

$$\text{and hence also } I = \frac{m \times l}{a \times l} = \frac{M}{V} = \frac{\text{Magnetic Moment}}{\text{Volume}}$$

Now a pole of strength m radiates $4\pi m$ lines of force (page 20) and hence the number of lines of force per unit area of cross-section will be $\frac{4\pi m}{a}$ or $4\pi I$, where I is the intensity of magnetisation.

Suppose a rod of iron or steel is placed in a magnetising field of strength H, and let the intensity of the induced magnetisation in the metal be I. Then there will be correspondingly $4\pi I$ lines of force per sq. cm. of cross-section in addition to the H lines of force per sq. cm. due to the field alone, i.e. Total Magnetic Induction (B) = Magnetising Field (H) + Induced Field ($4\pi I$) or $\mathbf{B = H + 4\pi\, I.}$

The **permeability** (μ) of the material is defined by the ratio B/H and the **susceptibility** (κ) by the ratio I/H.

The permeability μ varies with the strength H of the magnetising field, as becomes evident from the shape of the B—H curve (Fig. 83) for a specimen of iron or steel. The method of obtaining this curve can be described only briefly here. The specimen under test is placed in a solenoid, and a gradually increasing current is passed through the windings so that the specimen becomes magnetised. From the deflection of a magnetometer situated near one end of the solenoid, together with other ascertainable data, the total magnetic induction may be calculated.

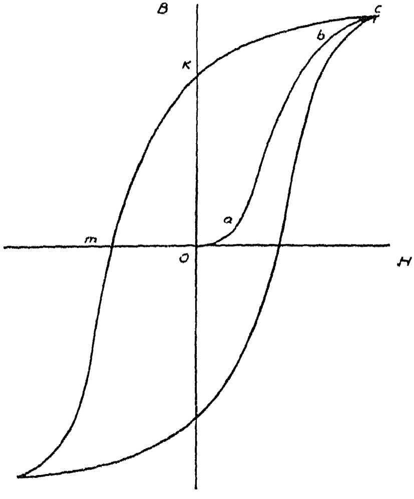


Fig. 83.

On inspection of Fig. 83 it is evident that for small values of H the magnetic induction B increases slowly, as shown by O*a* of the curve. From *a* to *b* the induction increases very rapidly, but when H has attained a certain value, the curve gradually becomes a straight line (*bc* in figure) slightly inclined to the axis of H. This last stage of the curve represents the condition of **magnetic saturation** of the iron, and serves as an effective illustration of the molecular theory of magnetism (page 11).

In the above experiment after the specimen has become saturated, suppose that (1) the current is gradually reduced until its value is zero, (2) it is reversed and its value increased until the saturation stage is again reached, (3) it is diminished to zero, and (4) the current is again reversed and gradually increased until the specimen is saturated. The curve (Fig. 83) obtained by plotting various corresponding values of B and H represents a **cycle of magnetisation** for the specimen, and it shows that the magnetic induction *lags* behind the magnetising force. On this latter account the curve is generally known as a *hysteresis* loop ("hysteresis" being derived from a Greek word which means "to lag behind"). The area of this loop is a measure of the work done in carrying out the cycle of magnetisation, and this energy is dissipated as heat, being greater per unit volume in the case of steel than for soft iron. Consequently, in electrical machinery, the parts that are subjected to rapid reversals of current are usually made of iron to reduce the heating effect.

Referring to Fig. 83 it is seen that when the magnetising force is reduced to zero, after saturation has first been reached, the *residual magnetic induction* or **remanent magnetism** is represented by OK. Further, a reversing field of value O*m* has to be applied before the magnetic induction becomes zero. This value of field strength O*m* is called the **coercive force** for the particular substance.

In recent years a number of alloys have been discovered which have a greater coercive force than ordinary steel, e.g. cobalt steel which is employed in permanent magnet moving-coil loud speakers. On the other hand nickel alloys are employed when low hysteresis and high permeability are required, such as in the construction of transformers.

MAGNETIC CLASSIFICATION OF SUBSTANCES.—In 1845, Faraday, by using powerful electro-magnets, showed that all bodies are susceptible to magnetic influence, but that they do not all behave alike under this influence. He showed further that substances can be divided into three classes according to their magnetic characteristics, viz. (1) ferromagnetic substances, (2) paramagnetic substances, and (3) diamagnetic substances.

(1) *Ferromagnetic Substances.*—Iron, steel, cobalt, nickel and certain alloys belong to this class. If these substances are placed in a magnetic field they set themselves with their axes in the direction of the field, i.e. they *tend to move from weaker parts of the field to stronger parts.*

(2) *Paramagnetic Substances.*—These include the metals platinum and osmium. Oxygen gas is also a paramagnetic substance. These substances *behave like ferromagnetic substances* except that they become much more feebly magnetised than the latter.

(3) *Diamagnetic Substances.*—Examples of diamagnetic substances are bismuth, mercury, zinc and lead. These

substances are repelled by a magnet. If a diamagnetic substance is suspended in a magnetic field it tends to set itself perpendicular to the field, i.e. *it tends to move from the stronger to the weaker parts of the field.*

EXERCISES.

1. State briefly what the letters H, V, and I signify, as used in connection with terrestrial magnetism. Draw a figure of three lines showing the application of these letters to the three lines of the figure. The lines should be drawn, as regards direction, in their proper relationship to each other, when considered as representing magnetic forces. The figure should be drawn upon the assumption that the north and south geographical poles are, respectively, points at the centres of the top and bottom edges of a page of your examination paper. Connect these two points by a straight line. Show in their proper places, the angles of declination and inclination (dip), and their approximately correct values, in degrees, for the present year at Greenwich. (C. & G., 1926.)

2. Explain briefly the terms (*a*) Magnetic Meridian, (*b*) Dip and (*c*) Declination.

Describe an experiment by which the values of the horizontal intensity of the earth's field at two different points may be compared. (U.E.I., S1, 1933.)

3. If a magnet makes 12 complete vibrations in 90 seconds at a place where the field strength is 0·2 gauss, find its period at a place where the strength of the field is 0·15 gauss.

4. Define the magnetic moment of a magnet.

A magnet of mass 25 gms. is 10 cms. long and 1 cm. broad. If it is set oscillating in a field of strength 0·17 gauss it performs 20 oscillations in 3 minutes. Find the magnetic moment of the magnet. (For a bar-magnet $I = \frac{m}{12}(l^2 + b^2)$, where m is the mass, l the length and b the breadth of the magnet respectively.)

5. Describe the Tangent Magnetometer. How may it be used to verify the Inverse Square Law in Magnetism?

6. What do you understand by the terms Intensity of Magnetisation, Permeability, and Susceptibility?

7. The current through the coils of an electro-magnet is made to rise gradually from nothing to, say, 10 amps. Draw a curve to show the manner in which the magnetic flux (symbol = B) through the iron changes, as the magnetising field (symbol = H) due to the current increases. Explain the reasons for the curve assuming the form that you show. Indicate on the curve the point of saturation of the iron, giving your reasons. (C. & G., 1925.)

8. Describe the effect known as hysteresis obtained when iron is carried through a cycle of magnetisation.

Why is this effect undesirable in the armature core of a dynamo? (U.L.C.I., B, 1935.)

9. Explain briefly the meaning of "permeability." Two similar circular rings of soft iron are magnetised, the first by being placed between the poles of a strong horseshoe magnet, so that a diameter of the ring is in line with the poles of the magnet, the second by means of a coil of wire wound on the ring through which a strong current flows from a battery.

Show in each case, by means of sketches, the lines of magnetic force around the ring. (C. & G., 1931.)

CHAPTER VI

RESISTANCE: ELECTRIC CIRCUITS

RESISTANCES IN SERIES.—Resistance is the opposition offered to the passage of an electric current through a substance, and if the same current passes successively through each of a number of resistances they are said to be connected in series. It is evident that the *total resistance will be the sum of the separate resistances*, but a formal proof may be deduced from Ohm's Law as follows: Let R_1, R_2 and R_3 ohms be three resistances connected in series (Fig. 84), and V_1, V_2 and V_3 volts be the corresponding P.D.'s across the conductors when a current of I amps is passing.

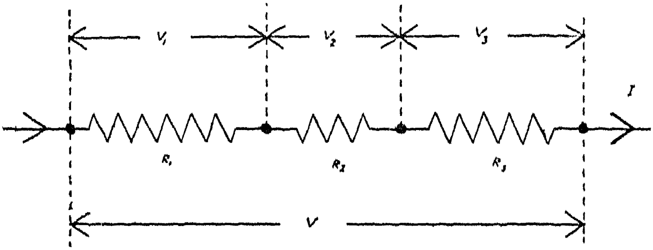


FIG. 84.

Applying Ohm's Law in turn to each conductor and then to the complete circuit, the following results are evident:

(1) $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$, and
(2) $V = IR$ where R is the composite resistance.

$$\text{But } V = V_1 + V_2 + V_3,$$

$$\text{i.e. } IR = IR_1 + IR_2 + IR_3$$

$$\text{Hence } \mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3.$$

RESISTANCES IN PARALLEL.—Fig. 85 shows three resistances R_1, R_2 and R_3 connected **in parallel,** or, as is sometimes referred to, *in multiple arc*. Now by Ohm's Law for a *given* P.D.

applied to the ends of any conductor the current passing along it will be $\propto \frac{1}{R}$, where R is the resistance of the conductor. Alternatively it may be said that the current is $\propto K$, which is defined as the **conductance** of the conductor. It is clear (Fig. 85) that the sum of the electric currents proceeding *away from* A must equal the total electric current moving *towards* A, otherwise there would be an accumulation of electricity at this point. This result is self-evident if the hydrostatic analogy is considered, where water flow corresponds to the electric current and water-pipes take the place of resistances. The law deduced above is due to Kirchhoff, a German scientist. It is known as *Kirchhoff's First Law*, and may be stated as follows : The *algebraic* sum of the currents meeting at any point in a network of conductors is zero.

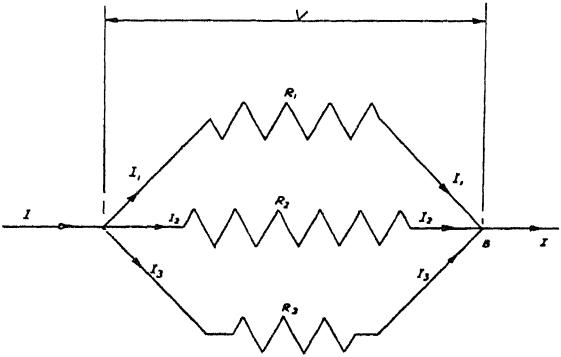


FIG. 85.

The term algebraic indicates that we must take into consideration the direction in which the current flows with respect to the point. Thus in Fig. 85, if a current I enters the network at A it distributes itself between the three branches of the network. If the values of the three currents are given by I_1, I_2 and I_3, then by Kirchhoff's First Law

$$I = I_1 + I_2 + I_3$$

Let V denote the potential difference between A and B, and let R be the *equivalent* resistance of the network. Since I is the total current flowing through the network it follows, by Ohm's Law, that $I = \frac{V}{R}$.

Similarly for resistance R_1, $I_1 = V/R_1$
,, ,, ,, R_2, $I_2 = V/R_2$
and ,, ,, R_3, $I_3 = V/R_3$

But $I = I_1 + I_2 + I_3$

$$\therefore \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\text{i.e. } \frac{1}{\mathbf{R}} = \frac{1}{\mathbf{R}_1} + \frac{1}{\mathbf{R}_2} + \frac{1}{\mathbf{R}_3}$$

This result may be extended to any number of resistances in parallel and may be stated thus: *The reciprocal of the equivalent resistance of a number of resistances in parallel is equal to the sum of the reciprocals of the individual resistances.*

Again it follows from the definition of conductance that $\mathbf{K} = \mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3$ where K is the equivalent conductance, and K_1, K_2 and K_3 are the conductances of the separate conductors. Hence *the combined conductance of a number of conductors in parallel is equal to the sum of their separate conductances.*

Example 1.—Three resistances, each of 6 ohms resistance, are connected in parallel. Find the equivalent resistance. Let R be the equivalent resistance.

$$\text{Then } \frac{1}{R} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore R = 2 \text{ ohms.}$$

Note.—If any number, n, of resistances each of which is equal to r ohms are connected in parallel, the equivalent resistance is equal to $\frac{r}{n}$ ohms.

Example 2.—Find the equivalent resistance of four coils whose resistances are 3ω, 6ω, 8ω and 15ω respectively, when they are connected in parallel. Let R be the equivalent resistance.

$$\text{Then } \frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{8} + \frac{1}{15}$$

$$= \frac{40 + 20 + 15 + 8}{120} = \frac{83}{120}$$

$$\therefore R = \frac{120}{83} = 1\frac{37}{83} \text{ ohms.}$$

N.B.—When any number of coils are connected in parallel, the resistance of the group must clearly be less than that of the smallest coil.

Example 3.—Calculate the resistance of the circuit shown in Fig. 86.

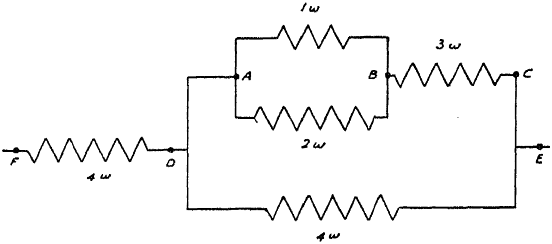


FIG. 86.

The resistance between A and B is equal to R_{AB} where

$$\frac{1}{R_{AB}} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}\ \omega, \text{ i.e. } R_{AB} = \frac{2}{3}\ \omega.$$

$\therefore$ Total resistance R_{AC} between A and C is equal to

$$R_{AB} + R_{BC} = \frac{2}{3} + 3 = \frac{11}{3}\omega.$$

Now the resistance (R_{DE}) between D and E will be given

$$\text{by} \quad \frac{1}{R_{DE}} = \frac{1}{R_{AC}} + \frac{1}{4} = \frac{3}{11} + \frac{1}{4}$$

$$\text{i.e. } R_{DE} = \frac{44}{23}\omega.$$

Hence the total resistance of the circuit is

$$R_{FE} = R_{FD} + R_{DE}$$
$$= 4 + \frac{44}{23}$$
$$= 5\frac{21}{23}\ \omega.$$

DISTRIBUTION OF CURRENTS IN A NETWORK.—Let a current I enter a network which consists of two coil resistances R_1

and R_2 connected in parallel as shown in Fig. 87. The main current subdivides and let I_1 be the current passing through the resistance R_1, while I_2 is that passing through the resistance R_2. Let V be the potential difference between A and B.

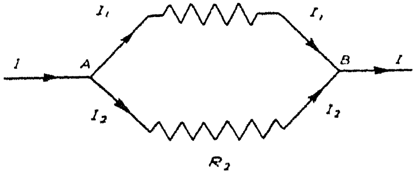


Fig. 87.

Then, applying Ohm's Law to the resistance R_1, $V=I_1R_1$, and applying Ohm's Law to the resistance R_2, $V=I_2R_2$

$$\therefore I_1R_1=I_2R_2 \text{ or } \frac{I_1}{I_2}=\frac{R_2}{R_1}$$

This shows that the currents in the two coils are inversely as the resistances of the coils.

Example.—The terminals of a battery of e.m.f. 2.2 volts and internal resistance 3 ohms are connected to two coils of resistances 4 ohms and 8 ohms respectively which are arranged in parallel. Find (1) the current through the battery, and (2) the current through each of the coils.

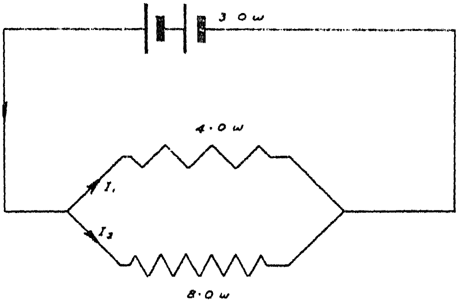


Fig. 88.

Let I be the current flowing through the battery. Also let I_1 be the current flowing through the 4 ohm coil and I_2 that flowing through the 8 ohm coil (Fig. 88).

If R is the equivalent resistance of the two coils,

$$\text{Then } \frac{1}{R}=\frac{1}{4}+\frac{1}{8}=\frac{2+1}{8}=\frac{3}{8}$$

$$\therefore R=\frac{8}{3}=2\frac{2}{3}\,\omega.$$

Now B the battery resistance is 3ω.

$$\text{Hence } I=\frac{E}{B+R}=\frac{2{\cdot}2}{3+8/3}=\frac{33}{85}\text{ amp.}$$

$$\text{Also } \frac{I_1}{I_2}=\frac{8}{4}=2 \quad \therefore\ I_1=2I_2$$

$$\text{Further } I_1+I_2=I=33/85 \text{ amp.}$$

On substitution $2I_2+I_2=33/85$, or $I_2=11/85$ amp. and $I_1=22/85$ amp.

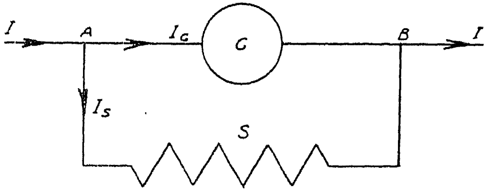


FIG. 89.

SHUNTS (see also page 108).—When it is desired to use a sensitive ammeter to measure a current larger than that given by the maximum reading, it is possible to do so by diverting a part of the main current from the instrument. This is accomplished by connecting a resistance in parallel with the ammeter, and the arrangement is comparable to the use of a "by-pass" road to avoid the traffic congestion in towns. The use of a shunt, as the parallel resistance is termed, is also adopted when a sensitive galvanometer is inserted in a circuit carrying a current of unknown magnitude, to prevent damage to the instrument. In this latter use the value of the shunt resistance would be adjusted to the needs of the experiment.

In Fig. 89 a galvanometer of resistance G is shown with a shunt resistance S.

At the point A, the current I divides into two parts, I_G which flows through the galvanometer, and I_S which flows through the shunt.

$$\text{Then } I = I_G + I_S.$$

Let E be the potential difference between the points A and B. Then, applying Ohm's Law (1) to the arm containing the galvanometer and (2) to the arm containing the shunt, we get

$$I_G \times G = E \quad . \quad . \quad . \quad . \quad (1)$$

$$I_S \times S = E \quad . \quad . \quad . \quad . \quad (2)$$

$$\therefore I_G \times G = I_S \times S,$$

$$\textbf{or } \mathbf{I_G = \frac{S}{G} \times I_S}.$$

From this it is evident that by making S small enough the fraction of the current flowing through the galvanometer can be reduced to any extent that is required.

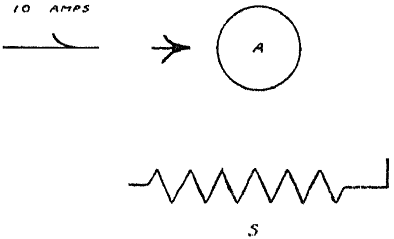


FIG. 90.

Example.—An ammeter of 1 ohm resistance is graduated to read 1 ampere. It is required to use the instrument in a circuit which carries 10 amperes. With what resistance must it be shunted ?

The ammeter is constructed to take only 1 ampere. Hence, when it is placed in a circuit through which 10 amperes flow, 9 amperes must be diverted from it. Hence (Fig. 90), if S is the resistance of the shunt

$$\begin{aligned} 9 \times S &= 1 \times \text{Resistance of ammeter} \\ &= 1 \times 1 \\ &= 1 \end{aligned}$$

$$\therefore S = \frac{1}{9} \text{ ohm.}$$

Example.—A galvanometer has a resistance of 198 ohms. With what resistance must it be shunted in order that it may take only $\frac{1}{100}$ of the current in a circuit in which it is placed ?

Let I denote the current in the main circuit (Fig. 91). If the current through the galvanometer is $\frac{I}{100}$, then the current that flows through the shunt is $\frac{99I}{100}$.

The resistance of the galvanometer is 198ω. Let S be the resistance of the shunt and let E be the potential difference between A and B.

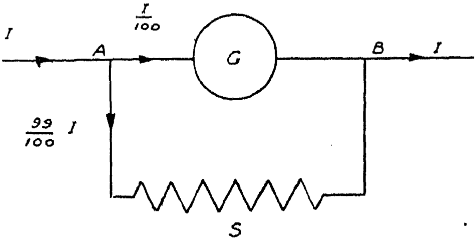


FIG. 91.

Applying Ohm's Law (1) to the arm containing the galvanometer and (2) to the arm containing the shunt, we get

$$\frac{I}{100} \times 198 = E$$

$$\text{and} \quad \frac{99I}{100} \times S = E$$

$$\therefore \frac{99I}{100} \times S = \frac{I}{100} \times 198$$

$$\therefore S = 2 \text{ ohms.}$$

It can be seen from this example that if an instrument is to take $\frac{1}{100}$th of the current in the circuit in which it is placed, the resistance of the shunt used must be $\frac{1}{99}$th of the resistance of the instrument. Likewise it can be shown that: If an instrument takes $\frac{1}{10}$th of the main current the resistance of the shunt is $\frac{1}{9}$th resistance of the instrument.

One instrument can clearly be used over several ranges when it is furnished with appropriate shunts.

Example.—A galvanometer of resistance 100 ohms is shunted with a coil whose resistance is 25 ohms. What is the combined resistance of the galvanometer and shunt? If the shunted galvanometer is connected to a battery of e.m.f. 40 volts and internal resistance 20 ohms, what is the current (1) through the battery, and (2) through the galvanometer?

The combined resistance of the galvanometer and shunt is given by R,

$$\text{where } \frac{1}{R}=\frac{1}{100}+\frac{1}{25}=\frac{5}{100}$$

$$\therefore R=20 \text{ ohms.}$$

Let I be the current through the battery, E its e.m.f. and B its internal resistance.

$$\text{Then } I=\frac{E}{B+R}=\frac{40}{20+20}=1 \text{ ampere.}$$

Let I_G = current through the galvanometer
and I_S = current through the shunt.

$$\frac{I_S}{I_G}=\frac{100}{25}=4 \text{ or } I_S=4\,I_G$$

$$\text{Also } I_S+I_G=I$$

$$\therefore 4I_G+I_G=1,$$

$$\text{or } 5I_G=1, \text{ i.e. } I_G=0.2 \text{ ampere.}$$

Cells in Series and in Parallel.—When cells are connected in series the positive terminal of one cell is joined to the negative terminal of the adjacent cell. In the parallel arrangement the positive terminals are connected together to form one plate of the combined battery and, likewise, the negative terminals are connected together.

When n cells, each of electro-motive force E volts and internal resistance B ohms are connected in series the total electro-motive force is nE volts, and the total internal resistance is nB ohms. If such a battery is connected to an external resistance R ohms (Fig. 92 (*a*)), the current in amperes flowing through the circuit is given by

$$I=\frac{nE}{nB+R} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Before considering the case of the parallel connection of cells it should be observed that the electro-motive force of a cell is independent of the size of the plates. The latter, however, does influence the resistance of the cell, for, just as a wide tube or pipe conveys more water in a given time than does a narrow one, so an increase in the area of the plates of a cell increases the area of the channel through which the current flows and hence diminishes the resistance. Quantitatively, if n cells each of e.m.f. E volts and internal resistance B ohms are connected in parallel, the total e.m.f. is E volts, and the total internal resistance is $\frac{B}{n}$ ohms. If such a battery is connected through an external resistance R ohms as shown in Fig. 92 (b), the current through the external circuit is given by

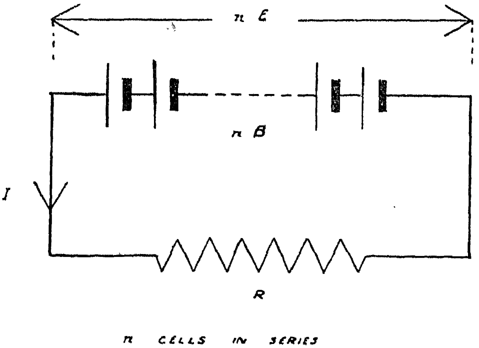


FIG. 92 (a).

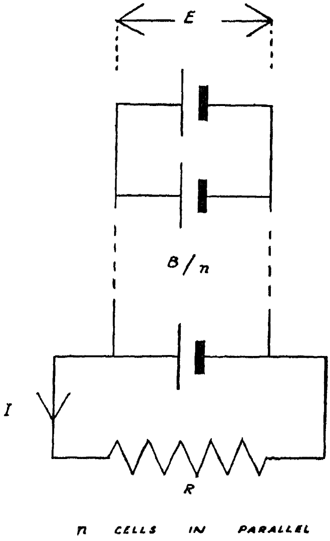


FIG. 92 (b).

$$I = \frac{E}{\frac{B}{n} + R}$$

$$= \frac{nE}{B + nR} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Example.—(*a*) The terminals of a Leclanché cell of e.m.f. 1.4 volts and internal resistance 3·2 ohms are connected to a coil whose resistance is 1 ohm. Find the current flowing through the coil.

(*b*) Find also the current when five such Leclanché cells are connected in series and used to send a current through the same coil.

(*c*) Work out also the value of the current when the cells are connected in parallel.

These questions can be solved by the direct application of Ohm's Law. Using notation as above, we get for

$$(a) \quad I = \frac{E}{B+R}$$

$$= \frac{1{\cdot}4}{3{\cdot}2+1} = \frac{1{\cdot}4}{4{\cdot}2} = \frac{1}{3} = {\cdot}33 \text{ amp.}$$

In the case of (*b*)

$$I = \frac{nE}{nB+R}$$

$$= \frac{5 \times 1{\cdot}4}{5 \times 3{\cdot}2 + 1} = \frac{7}{17} = {\cdot}41 \text{ amp.}$$

In the case of (*c*)

$$I = \frac{nE}{B+nR}$$

$$= \frac{5 \times 1{\cdot}4}{3{\cdot}2+5} = \frac{7}{8{\cdot}2} = {\cdot}85 \text{ amp.}$$

It can be seen that the introduction of four additional cells in the series arrangement results in an increase of only ·08 ampere in the current. When, however, the cells are connected in parallel, the current furnished by the five cells is 2·6 times as great as that supplied by the single cell.

It is found that when the internal resistance is high by comparison with the external resistance, very little advantage is gained by connecting the cells in series. The parallel arrangement is more suitable. When the internal resistance is low by comparison with the external resistance the series arrangement is the better. The student can test this statement further by working the following example :

Example.—(*a*) The terminals of a cell of e.m.f. 2 volts and internal resistance ·2 ohm are connected to a coil whose resistance is 20 ohms. Find the current flowing through the coil.

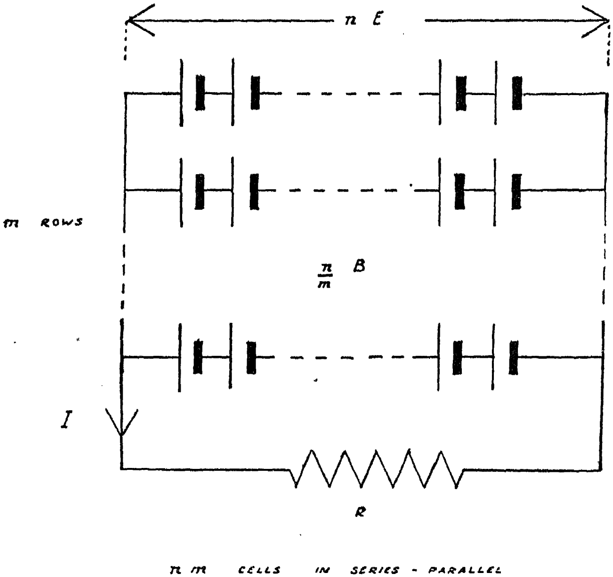


FIG. 93.

(*b*) Find also the current when five such cells connected in series are used to send a current through the same coil.

(*c*) Determine also the value of the current when the cells are connected in parallel.

(Answer.—(*a*) ·10 amp. approx. (*b*) ·48 amp. approx. (*c*) ·10 amp. approx.)

ARRANGEMENT OF CELLS FOR MAXIMUM CURRENT IN A CIRCUIT.—Consider a mixed grouping of $m \times n$ cells (Fig. 93) where n cells are connected in series in a row and there are m rows in parallel.

Let E be the e.m.f. and B the battery resistance of each cell respectively, and let R be the external resistance.

Then the total resistance of *each* row of cells is $n \times B$, and therefore the resistance of m rows in parallel is $\frac{n.B}{m}$

But the total e.m.f. of the cells as connected is $n.E$. Hence the current in the main circuit is

$$I = \frac{\text{Total e.m.f.}}{\text{Total Resistance}} = \frac{nE}{R + \frac{n.B}{m}}$$

Alternatively this expression may be written as

$$I = \frac{E}{\frac{R}{n} + \frac{B}{m}}$$

It is now evident that I will be a maximum when $\left(\frac{R}{n} + \frac{B}{m}\right)$ is a minimum, since E is a constant.

Mathematically it may be shown that this condition will occur if $\frac{R}{n} = \frac{B}{m}$

$$\text{i.e. if } R = \frac{nB}{m}$$

But $\frac{nB}{m}$ is the equivalent resistance of the **battery** of cells, and R is the external resistance of the circuit. Hence in order to obtain the maximum current in a circuit, the arrangement of cells should be such that *the internal resistance of the battery is as nearly as possible equal to the external resistance.* It should be noted that, in this particular grouping of the cells, half the available energy is used in the battery itself, an uneconomical process.

Example.—A battery consisting of 36 cells, each of resistance 3ω, is available for heating a coil of resistance 12ω. This coil is immersed in a liquid which it is required to heat as rapidly as possible, so that the problem is to determine the arrangement of cells which will provide the maximum current in the resistance.

Here $B=3\omega$, $R=12\omega$ and $n\times m=36$. Now the maximum current will be obtained when $\frac{R}{n}=\frac{B}{m}$

$$\text{i.e. } \frac{m}{n}=\frac{B}{R}=\frac{3}{12}=\frac{1}{4}$$

$$\text{But } mn=36 \text{ or } \frac{m}{36}=\frac{1}{n}$$

$$\therefore \frac{m}{n}=\frac{m^2}{36}=\frac{1}{4}$$

$$\text{or } m^2=9$$

$$\text{Hence } m=3 \text{ and } n=12.$$

The battery should therefore be connected so that there are three rows of cells in parallel, each row containing 12 cells.

The current passing in the resistance will then be given by $I=\frac{E}{\frac{R}{n}+\frac{B}{m}}=\frac{E}{1+1}=\frac{E}{2}$ amps., where E is the e.m.f. (in volts) of a single cell.

Great care should always be taken to avoid an arrangement of cells which would result in any one of them delivering an excessively large current. In such circumstances the cell would be ruined by the large development of heat, and by the loosening of the material in the grids of the plates.

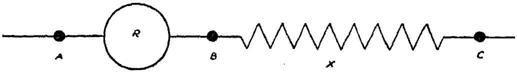


FIG. 94.

SERIES RESISTANCE OF A VOLTMETER.—The moving coil system of an ammeter (page 185) may be employed as a voltmeter, by the addition of a series resistance in the base of the instrument. Furthermore the range of the voltmeter may be varied by the adjustment of the value of this series resistance. As an example consider a moving coil system whose sensitivity is such as to give a full scale deflection for a current of 0·02 ampere. In order to use this system as a voltmeter to measure P.D.'s up to 300 volts, a series resistance X (Fig. 94) must be included. X is chosen so that a current of 0·02 amp. passes, and a full scale deflection is obtained when 300 volts is applied across the remote ends A and C of the coil R and X respectively.

By Ohm's Law, Current × Resistance = P.D.

$$\therefore 0{\cdot}02 \times (R+X) = 300$$
$$\text{or } R+X = 15{,}000\omega.$$

But if $R=100\omega$, then $X=14{,}900\omega$. Similarly for a range of 500 volts, $R+X=25{,}000\omega$, so that the series resistance will be $24{,}900\omega$.

Example.—A micro-ammeter of 49·5ω resistance has a maximum reading of 120 microamps. What resistance must be inserted in series with this instrument so that it can be utilised to measure a P.D. of 15 millivolts?

Using the notation of Fig. 94, $R=49{\cdot}5\omega$. Now a full-scale deflection is produced by a current of 120×10^{-6} amps.

Hence by Ohm's Law

$$\text{Current} \times \text{Resistance} = \text{P.D.}$$
$$\text{and } \therefore 120 \times 10^{-6} (R+X) = 15 \times 10^{-3}$$
$$\text{or } (R+X) = 125\omega.$$
$$\text{i.e. } X = 75{\cdot}5\omega.$$

Pressure and Resistance.—If two conductors are placed together in light contact, the irregularities of their surfaces may be such that only a small fraction of their areas are close enough together to permit a flow of electrons from one to the other. On pressing together the conductors their "conducting area" may be found to increase by as much as ten times its previous value, and hence if a contact of this nature is included in an electrical circuit, the current flowing will vary with the applied pressure. This effect will be most pronounced when the "contacting" conductors are composed of a comparatively poor conducting material, such as carbon, and are included in a low resistance circuit.

An important application utilising the above effect is a device used in telephony known as the **microphone.** In principle it consists of a number of carbon granules contained in a chamber between two carbon plates, C_1 and C_2 (Fig. 95 (*a*)) which are included in an electrical circuit as indicated. A suitable diaphragm D is fixed to the movable electrode C_1, so that impinging sound waves cause it to vibrate and subject the granules to a varying pressure. The resistance between the electrodes will consequently vary, and the current in the circuit will fluctuate in unison with the sound waves. Hence the action of speaking in front of the diaphragm causes the current in the circuit to vary or to be "modulated," as it is termed, so that by this means speech can be transmitted over long distances (page 257). Carbon is very suitable for the inter-electrode material since it is a poor conductor and cheap, and also as it does not oxidise, the characteristics of the instrument remain fairly constant.

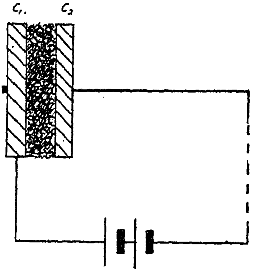


Fig. 95 (*a*).

The Siemen's type of transmitter as used in a hand micro-telephone is shown in section in Fig. 95 (*b*). The diaphragm D consists of a single aluminium cone, which is corrugated to give increased strength, and attached to its centre is a small aluminium cylinder A. The moving carbon electrode E_1 projects into the granule chamber C, and opposite to it is the second carbon electrode E_2, which is fixed to the base of the instrument casing. The electrodes are arranged carefully to be near the centre of the chamber with their polished faces about 0·08 in. apart. Leakage of carbon granules is prevented by the use of rings of silk clamped in the granule chamber and forming a sliding fit on the cylinder A. Both electrodes are always completely immersed in granules whatever the position of the transmitter. Electrical connections are respectively made by a plug fitting the socket S and by springs (not shown in Fig. 95 (*b*)) which make contact with the instrument case. The diaphragm is protected from damage by means of a perforated ebonite cover G. O is an oiled-silk cover.

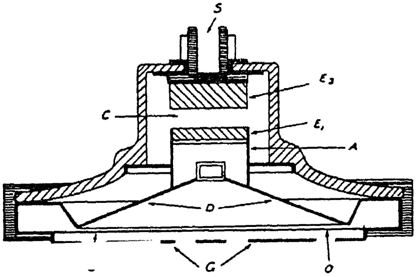


Fig. 95 (*b*).

Rheostats.—A general requirement of electrical circuits is the ability to vary the current, continuously or in steps, without breaking the circuit. The variable resistances which are used to regulate the current in this manner are known as rheostats, and their specification should include the maximum safe current which may be carried without overheating.

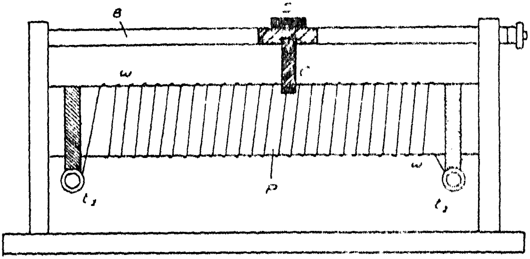


FIG. 96.

Carbon Rheostats.—Consist of a pile of carbon plates whose resistance may be altered by varying the contact pressure between the plates by means of a thumb screw. The ratio of the maximum to minimum resistance is approximately 2 to 1. Further adjustment is possible by altering the position of the *metal* terminal plates to change the number of carbon plates in the circuit. This type of resistance is useful for fairly large currents, and for a carbon rheostat of, say, 8 ohms maximum resistance the maximum safe current would be of the order of 6 amps.

Wire Resistances.—Fig. 96 shows a common type of variable wire resistance comprising a number of turns of wire, of a copper-nickel alloy, wound on a hollow insulating tube P. This wire has a surface oxidised layer which effectively insulates

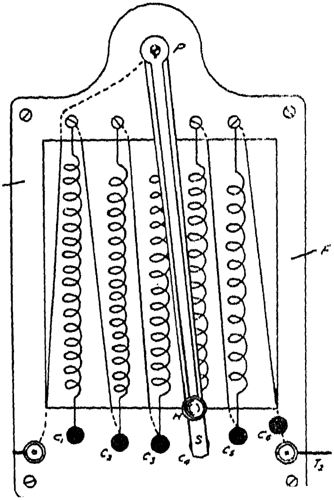


FIG. 97.

the turns from one another, and the length included in the electrical circuit is varied by the movement of a slider S. This slider makes contact between the wire at C and a horizontal metal rod B which carries a terminal T at one end. The external circuit connections are made at T and at *either* of the two bottom terminals t_1 and t_2. The cooling of these resistances is most effective when they are used in a vertical position.

Another variety of wire resistance suitable for the rough regulation of currents up to, say, 20 amps. is shown in Fig. 97. The number of wire spirals introduced into the circuit is varied by moving the switch arm S (H being an insulating handle), over the metal contact studs, C_1, C_2, etc., which are connected to the various resistance coils and are insulated from the metal frame F, by mica or slate. The switch arm is pivoted at P, and T_1 and T_2 are terminals.

Another very convenient and cheap form of current regulator of the "step-by-step" type can be built up of a number of lamps, which can be connected in series or parallel. The series arrangement is used when small currents are desired, the requisite variation of resistance being effected by "short-circuiting" lamps, i.e. connecting terminals with low resistance conductor.

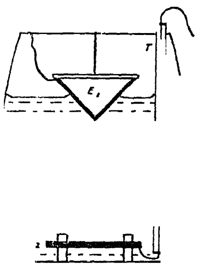


FIG. 98.

A type of variable liquid resistance used for starting or testing electric motors is shown in Fig. 98, where B is a barrel containing an electrolyte, which usually consists of an aqueous solution of common soda or sal-ammoniac. The lower fixed electrode E_2 is a metal plate, which is connected to the external circuit by a lead passing through an insulating tube T. The upper electrode E_1 is conical in shape, which permits of a gradual variation in resistance as it is lowered or raised into the solution by means of the balancing weight W.

For the construction of very large resistances of the order of megohms (10^6 ohms) either liquid or carbon resistances are employed. A convenient laboratory form of such a resistance is a pencil line drawn on a sheet of ebonite (or other insulating material), electrical contact being obtained at each end by strips of tin-foil firmly clamped under terminals. The common commercial type of high resistance is the grid leak which is used in wireless circuits. Cheap liquid resistances can be easily constructed using wooden troughs as containers and metal plates as *electrodes* (page 145). Suitable liquids are copper sulphate solution with copper electrodes, and zinc sulphate solution with electrodes of pure amalgamated zinc. Xylol is employed for very high resistances.

Appliances for making and breaking electric circuits.

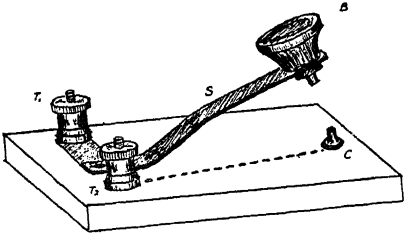


Fig. 99.

A *tapping key* (Fig. 99) is used when a circuit has only to be completed for a short time, as for example in the discharge of a condenser through a galvanometer (page 177). A key of this type is usually connected in *parallel* with the terminals of a moving-coil ballistic galvanometer, in order to damp (page 176) the motion of the moving system after deflection, and so bring it to rest as quickly as possible. The terminals T_2 and T_1 are connected to the metal stud C and to the spring S respectively. The circuit is only completed so long as the requisite pressure is applied on the ebonite button B. If the circuit is to be connected for an appreciable time period a plug key (Fig. 100) can be used. For the present purpose of making or breaking a circuit only two terminals, say A and B, are required, and these are connected to the free ends of the circuit. The circuit is completed by inserting a plug P in the hole F, the plug consisting of a conical piece of brass fixed in an ebonite head. The familiar tumbler switch as employed in domestic electric light installations may also be used as a "make" or "break" key.

A **commutator** is a device for reversing the current in any portion of a circuit without disconnecting any wires. Such a key must possess at least four terminals which are connected in pairs as indicated in Fig. 100, where it is desired to reverse the current through R. In this case the current will pass through the circuit in one direction when plugs are inserted in the holes F and H, and in the opposite sense on closing the gaps E and G. It should be carefully noted that the plugs are *never* inserted into adjacent holes.

Light and Resistance.—In some substances electrons can be removed from their atoms by the action of light and actually liberated from the surface of the body, giving rise to the phenomenon known as "photo-electricity." In other substances the energy acquired by the electrons is not sufficiently large to enable them to actually escape, but they exist in the material rather like the free electrons in a metal, so that the number of carriers of electric current is increased under the stimulus of light. In other words the resistance of the substance is decreased. The substance which shows this effect most markedly is selenium.

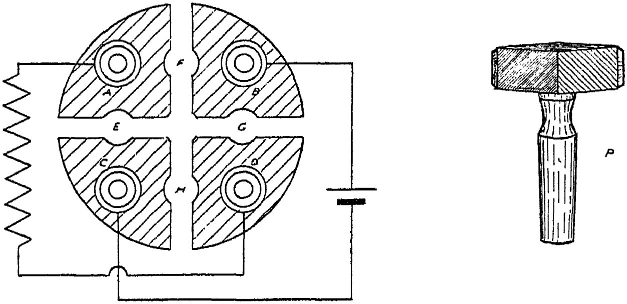


Fig. 100.

The commercial **selenium "cell,"** as it is unfortunately termed, consists of two long copper wire spirals, the wires running parallel to one another, and the space between them is filled with the grey modification of fused selenium. Such a " cell " is included in a circuit with a battery, and a " relay " which is usually an electro-magnet that will not operate until the current reaches a certain minimum value. The " dark " resistance of the selenium, say 300,000 ohms, is arranged to be sufficient to prevent any possibility of this minimum current flowing, but on exposure to light the resistance of the cell decreases and the relay will commence to function. The relay in turn is caused to operate suitable mechanism. By this means the switching " on " and " off " of lights at dusk and dawn respectively, can be controlled automatically.

The Magnetic Circuit.—The lines of force of a bar-magnet emerge from the north pole and, after passing through the air, re-enter the magnet at the south pole and continue as lines of induction in the iron. These closed curves define the so-called magnetic circuit and are analogous to lines of current flow. It must be emphasised, however, that there is *no* actual *flow* of anything in the magnetic circuit, whereas there is a definite flow of electrons in the case of an electrical circuit.

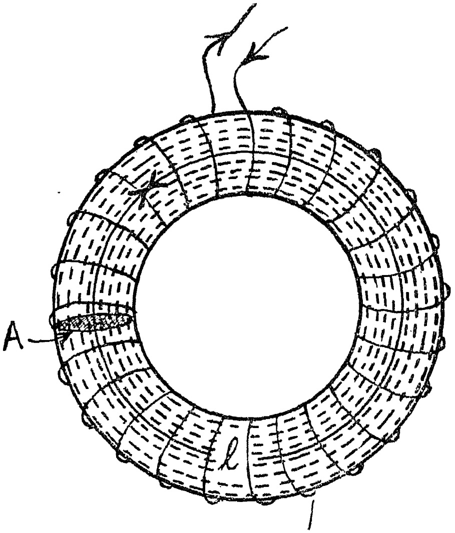


Fig. 101.

When a group of lines of magnetic force (or of induction) are considered the total number of lines is referred to as a **magnetic flux,** and is denoted by φ. The *flux density* is then given by φ / A where A is the area *perpendicular* to the direction of the flux.

In the simple case of an iron anchor ring (Fig. 101) uniformly wound with a magnetising solenoid of N turns, it can be shown that $\varphi = \frac{4\pi NI}{10} \Big/ \frac{l}{A\mu}$, where I (amps.) is the current flowing in the solenoid, l (cms.) is the mean total length of the magnetic circuit, A (sq. cm.) is the uniform cross-sectional area of the ring, and μ is the permeability of the iron.

Now Ohm's Law (page 65) states that current $=\frac{\text{electro-motive force}}{\text{resistance}}$ or $I=\frac{E}{R}$, and by analogy the following terms are used to specify a magnetic circuit. The *flux* φ is considered as analogous to an *electric current*, while the expression $\mathbf{4\pi NI/10}$ is known as the **magneto-motive force** (M.M.F.) which obviously corresponds to electro-motive force. It is evident by inspection of the expression cited above that the actual value of the flux produced by a given M.M.F. is determined by $\mathbf{l/A\mu}$, which is known as the **reluctance** of the magnetic circuit. The reluctance obviously compares with the resistance R of an electric circuit, which in the case of a rod conductor is given by $R=\rho\frac{l}{A}$ where l is the length and A the area of cross-section of the conductor. It is evident that $\frac{1}{\mu}$ corresponds to ρ the specific resistance. It should be carefully noted, however, that while ρ is independent of the value of the electric current flowing through the conductor, μ for a ferromagnetic material varies with the intensity of the magnetising field.

Summarising, then the law for the magnetic circuit is *Magnetic Flux=Magnetomotive Force÷Reluctance.*

Unit of M.M.F. is **Gilbert,** unit of FLUX is **Maxwell,** and unit of RELUCTANCE is **Oersted.**

Hence Maxwells=Gilberts÷Oersteds.

The practical examples of the magnetic circuit usually consist of an iron or steel portion with an intervening air-gap, as exemplified by the case of a motor or generator. Fig. 102 gives a diagrammatic representation of the magnetic circuit of a 4-pole generator. The latter consists of the magnet poles N. and S. fitted with soft iron shoes, and the armature C, which rotates in the gap between the shoes. The pole cores are surrounded by magnetising coils, while the armature itself comprises a soft iron core which is slotted to carry copper conductors (not indicated in Fig. 102). The path of the magnetic circuit is indicated by the dotted line U. If

l_1, l_2, l_3 and l_4 denote the lengths of the circuit, and a_1, a_2, a_3 and a_4 the corresponding mean areas of cross-section in the yoke, poles, air-gaps and armature, then the respective reluctances of the various portions of the circuit are given by $l_1/\mu_1 a_1$, $l_2/\mu_2 a_2$, $l_3/\mu_3 a_3$ and $l_4/\mu_4 a_4$, where μ_1, μ_2, μ_3 and μ_4 are the permeabilities of the different parts.

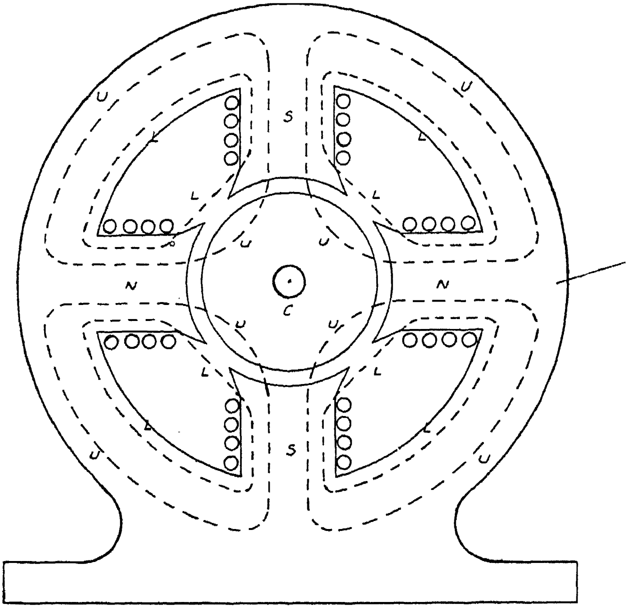


FIG. 102.

Now the total M.M.F=sum of products of Flux×Reluctance for the various portions of the magnetic circuit. Hence the total M.M.F.$=\varphi_1 l_1/\mu_1 a_1+\varphi_2 l_2/\mu_2 a_2+\varphi_3 l_3/\mu_3 a_3+\varphi_4 l_4/\mu_4 a_4$, where φ varies for each portion owing to a leakage of flux. In the diagram the stray flux L passes directly from pole to pole and thus avoids the path through the armature. In order to reduce this leakage to a minimum the air-gap between the poles and the armature is made as small as possible. The leakage factor for a given circuit is defined by the ratio total flux÷useful flux and in the case under consideration is given by (Flux U+Flux L)÷Flux U.

A recent application of the principles of the magnetic circuit is in radio-permeability tuning. This procedure consists in varying the inductance of a coil of wire (in order to tune a wireless circuit), by adjusting the size of the air-gap in an iron core.

Example 1.—A rectangular bar of iron 9 in. long, 1 in. wide and 0.6 in. thick has a reluctance of 0·0050 oersteds, when a uniformly distributed flux of 30,000 maxwells passes through the bar, in a direction parallel to the axis. Determine the flux density of the bar and its permeability to a flux of this density.

The flux density B (in gauss) is given by $\frac{\varphi}{A}$ where φ is the total flux, and A is the area of cross-section in sq. cms.

$$\text{Here } B = \frac{30{,}000}{1{\cdot}0 \times 0{\cdot}6 \times 2{\cdot}54^2} \text{ gauss}$$

$$= \mathbf{7752 \text{ gauss.}}$$

Now the reluctance $= \frac{l}{A\mu}$ where l is the length of the iron bar (in cms.) and μ is its permeability.

$$\text{Hence } \mu = \frac{l}{A} \div \text{reluctance}$$

$$\text{i.e. } \mu = \frac{9 \times 2{\cdot}54}{1{\cdot}0 \times 0{\cdot}6 \times 2{\cdot}54^2} \times \frac{}{0{\cdot}0050} = 1181.$$

Example 2.—Find the number of ampere turns needed to produce a flux of 40,000 maxwells through a closed soft iron circular ring of uniform cross-sectional area 4 sq. cms. The mean diameter of the ring is 20 cms., and the permeability may be taken as 1,000. How many additional ampere-turns will be required to obtain the same flux if an air-gap of mean thickness 1 mm. is cut in the ring ?

Using the previous nomenclature

$$\varphi = \frac{\dfrac{4\pi NI}{10}}{\dfrac{l}{A\mu}}$$

Hence the **number of ampere-turns** $= NI = \frac{\varphi l}{A\mu}\ \frac{10}{4\pi}$

$$\frac{40{,}000 \times \pi \times 20 \times 10}{4 \times 1000 \times 4\pi} = \mathbf{500}$$

In the second case the total reluctance of the circuit

$$= \left[\frac{\pi \times 20 - 0{\cdot}1}{4 \times 1000}\right] + \frac{0{\cdot}1}{4} = 0{\cdot}0407 \text{ oersteds.}$$

It is evident that even the presence of such a minute air-gap as the above has nearly trebled the reluctance of the circuit.

The total number of ampere-turns now required $= N^1I$

$$= \varphi \times \text{reluctance} \times \frac{10}{4\pi}$$

$$= \frac{40{,}000 \times 0{\cdot}0407 \times 10}{4\pi} = 1296$$

Hence **additional number** of ampere-turns required $=$ **796.**

QUESTIONS.

1. On what does the resistance of a wire depend ?

Four accumulators, each of 2 volts and of negligible internal resistance, are connected in series to three resistances, each of 50 ohms, also in series. Calculate the current in the circuit, and the potential difference between the ends of each resistance. (U.L.C.I., A, 1935.)

2. Three resistances of 12 ohms, 18 ohms, and 36 ohms respectively, are joined in parallel, and this group of three resistances is placed in series with a single resistance of 2 ohms and a 24 volt battery of negligible resistance.

Find the total current taken from the battery and the current passing through the 12 ohms resistance. What would be the values of the currents if the battery had an internal resistance of 4 ohms ? (C. & G., 1928.)

3. Four cells each of 1·5 volts and 0·5 ohm internal resistance are available, and it is required to send the maximum possible current through an electro-magnet having a resistance of 0·5 ohm. Should all the cells be joined in series or would it be better to join the cells in two parallel groups, each group consisting of two cells in series ? What would be the current in each case ? (C. & G., 1929).

4. A series field coil of a generator consists of 19 metres of copper wire of square section 0·5 cm. × 0·5 cm. Given that the specific resistance of copper is 1·7 microhms per cm. cube, calculate the resistance of the coil. (U.E.I., S1, 1932.)

5. Five similar electric lamps are grouped in parallel across a pair of leads connected to the terminals of a battery of 18 volts e.m.f. and 0·1 ohm resistance. The resistance of the leads to the point where the lamps are grouped is also 0·1 ohm. Draw a diagram of the arrangement. If the current is 0·1 amp through each lamp, calculate the resistance of each lamp when the current is flowing. (C. & G., 1927.)

6. Explain by formula, or otherwise, what is meant by Ohm's Law ; hence define the unit of resistance in terms of the volt and ampere.

A galvanometer of resistance 45 ohms has a shunt of 5 ohms. What is the combined resistance of the galvanometer with shunt ? What current will be taken from a battery of 100 volts having an internal resistance of

0·5 ohm, when the shunted galvanometer is connected across the terminals of the battery ? (C. & G., 1929.)

7. A voltaic cell, of 1 ohm resistance, is connected to a wire of 2 ohms resistance, which is in parallel with a certain number of wires, each of 50 ohms resistance. The total resistance of the circuit is 2 ohms. State the number of wires of 50 ohms resistance. (C. & G., 1927.)

8. Calculate the number of cells, each of 1·3 volts e.m.f. and negligible resistance, required to furnish a current of 50 milliamperes through an external resistance of 170 ohms. (C. & G., 1924.)

9. Explain the action of a shunt.

It is necessary to use an ammeter which is graduated up to one ampere to measure currents up to 10 amperes. If the resistance of the ammeter is 0·5 ohm with what resistance must the instrument be shunted ?

10. What is the purpose of a voltmeter ? How may the range of a voltmeter be increased ?

It is desired to use a 0·2 voltmeter having a resistance of 500 ohms as a voltmeter reading to 100 volts. What must be the resistance of a coil placed in series with the voltmeter, so that the instrument may read 100 volts for full-scale deflection ? (U.L.C.I. 1929).

11. An iron ring has a mean circumference of 45 cm. and a cross-sectional area of 5 sq. cms. It is uniformly wound with 200 turns. When the exciting current is 1 amp., the flux in the ring is 24,000 lines. Calculate (*a*) the permeability of the iron ; (*b*) the current required to establish the same flux when a radial air gap of 0·2 cm. is made in the ring. (U.E.I., S2, 1935.)

12. A metal ring of 25 cm. mean diameter is wound uniformly with 500 turns and a current of 6 amp. passed through the coil. The cross-sectional area of the ring is 4 sq. cm. Calculate the magnetic flux established if the ring is (*a*) of copper, and (*b*) of steel working at 400 permeability. How does the permeability of iron depend upon the flux density ? (N.C.T.E.C., 1933.)

CHAPTER VII

HEATING EFFECTS OF THE CURRENT

When an electric current is passed along a metallic conductor, no change in the nature of the material is found in the neighbourhood of the points where the current enters or leaves (cf. electrolytes, page 143). It appears, therefore, that the conduction of electricity in such bodies takes place without any *material* carriers, and so it is inferred that only the electrons and *not* the +vely charged atoms move under the influence of an electric field. The precise nature of the mechanism of this electronic motion is still very uncertain, but roughly it may be supposed that the conductor can be regarded as a sponge, the electrons passing through its pores like the percolation of water.

By reason of their velocity, which is actually quite small, the electrons will possess kinetic energy and some of this energy will be imparted to the atoms of the substance with which they collide. Since an increase in the kinetic energy of the molecules of a substance corresponds to a rise of temperature, it follows that the electrical energy required to *maintain* the current will appear in the form of heat. The phenomena is analogous to the heat generated when a body overcomes frictional resistance during mechanical motion. The dependency of modern civilisation upon this heating effect of an electric current is quite evident, when one considers such examples as incandescent electric lamps, electric fires and heaters, etc.

Experiment.—Stretch a tin fuse wire horizontally between two terminals fixed in insulated supports. Join in series with the wire a battery, a key and a variable resistance. Adjust the resistance so that after closing the circuit for a short time the wire begins to sag. This effect is due to the expansion of the wire produced by the heat generated during the passage of the current.

If the rise of temperature in any conductor becomes too great the wire may eventually glow, and in this manner give rise to a fire and possible damage to property. Such a happening may result from a sudden increase of current due to a short circuit in the line, and to avoid this possibility fuse wires (see also page 198) are inserted in the circuit. These wires are chosen of a suitable diameter and material so that they will melt, and therefore break the circuit, when any current passes which is above that deemed to be safe for the particular circuit.

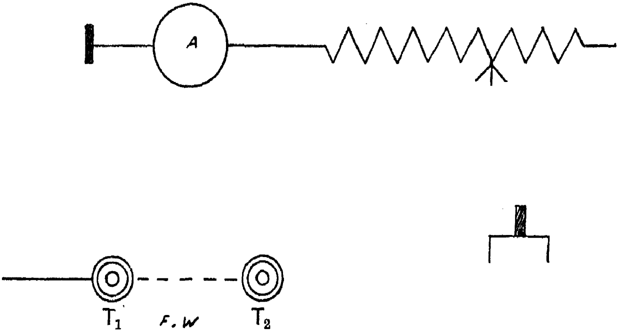


FIG. 103.

Experiment.—Connect a battery B, an ammeter A, a rheostat R and a key K in series, and insert also in the circuit a short length of fuse wire (F.W.) which "bridges" the gap between two terminals, T_1 and T_2, mounted on a baseboard (Fig. 103). Adjust the rheostat so that, at first, the maximum possible resistance is in the circuit. Gradually diminish the resistance, and continue doing this until the fuse "*blows*." Observe the ammeter reading when this actually happens.

Repeat, using other fuse wires of different gauges.

The fuse wires melt as the result of the heat produced by the passage of the current through them.

Tabulate the fusing current against the gauge for wires of different materials.

Wire of Material " A "

Current						
S.W.G.						

Wire of Material " B "

Current						
S.W.G.						

Experiment.—To determine the relation between fusing current and length of fuse wire. In this case the same material and diameter of wire is used throughout the experiment, but the length of wire is varied. In order to change the length of the wire one of the terminals on the baseboard should be movable. Make three or four determinations of the fusing current for *each* length of wire and tabulate your results as below :

Length of Wire (cms.).	Values of Fusing Current (amps.).	Mean Value of Fusing Current (amps.).

It should be noted that the fusing current is greater for the shorter lengths, since the cooling due to heat loss by thermal conduction at the ends of the wire is then more effective.

LAWS OF HEATING EFFECT.—A Manchester brewer named **oule** (1818-1889), who took up scientific research as a hobby, nvestigated the laws relating to the heating effects of a current. The results of his observations may be summarised as follows :

(1) The heat (H) produced in a conductor in a given time is $\propto$ square of the current strength (I), e.g. a current of two amps. generates four times as much heat in a conductor as does a current of one ampere flowing through the same conductor for the same time.

$$\text{i.e. } H \propto I^2$$

(2) The heat developed in a conductor in a given time by a given current is proportional to the resistance (R) of the conductor, e.g. a certain current will develop twice as much heat in a resistance of two ohms as it will in the same time in a resistance of one ohm.

$$\text{i.e. } H \propto R$$

(3) The heat developed in any conductor by a given current is proportional to the time (t) during which the current flows, e.g. a given current flowing through a conductor develops twice as much heat in the conductor in two seconds as it does in one second.

$$\text{i.e. } H \propto t$$

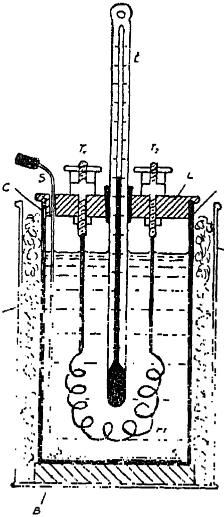


FIG. 104.

The form of calorimeter suitable for measuring the heat generated by an electric current is shown in Fig. 104. The manganin - heating coil H is soldered to copper leads which are in turn soldered to the terminals T_1 and T_2. These terminals are fixed to an ebonite lid L closing the top of the copper calorimeter C. A stirrer S and a thermometer t also pass through holes in the lid. In order to minimise loss of heat to the surroundings the calorimeter rests on an insulating block B, placed inside the wider vessel J, the interspace being loosely packed with cotton wool.

Experiment.—To show that $H \propto I^2$ for a given coil and given time. Connect a battery B consisting of about six accumulators,* a variable rheostat R, an ammeter A, and a key K in series with the coil H, and arrange the connections so that the coil can be immersed in a known weight of water contained in a calorimeter (as in Fig. 105 with voltmeter V removed from circuit). Place a thermometer in the water and pass a current of known strength through the circuit. Stir the water gently with the stirrer and observe the *rise* in the temperature of the water in a time *t* secs. Let this be θ_1°C.

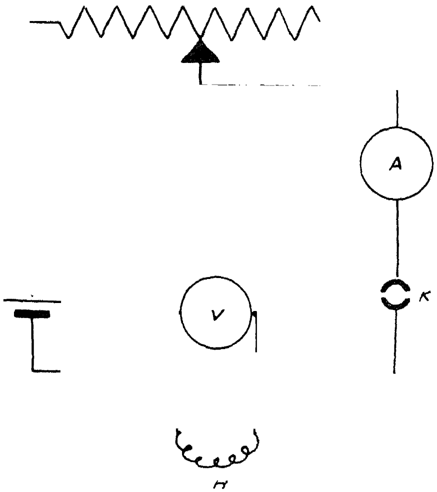


Fig. 105.

Pour the water out of the calorimeter and put the same weight of cold water in it again. Next, adjust the rheostat so that the current in the circuit has a value which is some multiple of the previous value. Suppose the current in the second experiment is twice as great as that in the first and let the rise of temperature now be θ_2°C.

$$\text{Then} \quad - = \frac{2^2}{1^2} = 4$$

i.e. the rise in temperature in the second case should be four times as great as in the first case.

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* D.C. Mains, if available, are a very suitable source of supply in this experiment.

If possible, take a number of values of rise of temperature for various values of the current. Tabulate your results as follows:

I (amp.)	I^2	θ°C.	$\frac{I^2}{\theta}$.

The last column should be constant. If θ is plotted against I^2, a straight line graph should be obtained.

As previously hinted at earlier in this chapter the heating effect of a current is of prime importance in the transmission of power by means of insulated cables, as if the temperature of the conductor becomes excessive the insulation will become impaired. In the case of rubber insulation, for example, a safe temperature is reckoned to be one below about 120° F.

In the matter of the wiring of dwelling houses two separate circuits are generally necessary, one for lighting and a thicker cable for heating, cooking, etc., otherwise the lighting wiring would become overheated by the larger currents required for the various domestic appliances. The need for two sets of wiring is also impelled by the fact that two meters are required to register separately the lighting and the power consumptions, as the rate of cost per unit of electricity is different in the two cases.

Experiment.—To show that, for a given current, $H \propto R$ in a given time. Use the same apparatus as in the last experiment. Observe the rise in the temperature of the water when a known current passes through the coil for a definite time. Next replace the coil with another of a different resistance, and by means of the rheostat adjust the current in the circuit to its previous value. Again observe the rise in temperature in the same time. Repeat, using a number of other coils. The increase of temperature in each case should be proportional to the resistance of the coil employed. If increase of temperature is plotted against resistance a straight line graph should be obtained.